I -440 ON THE SLIDING MODE CONTROL OF BUILDING WITH ATMD

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1. Introduction

Sliding Mode Control (SMC) is a powerful nonlinear control suitable for controlling structures with uncertainties associated with them, e.g. buildings excited by uncertain wind load or earthquake load. However, before making SMC applicable to practicle civil engineering control problems, many problems associated with the application of SMC have to be solved. In this paper one of such problems associated with the application of the SMC theory for controlling a building with ATMD is addressed and solved.

2. Sliding Mode Control of Buildings

In SMC framework, a non–linear switched feedback control law is obtained which drives the trajectories of the controlled system on to a specified surface, the *sliding surface* $\sigma(\mathbf{x})$, embedded in the state–space of the dynamical system, and maintains the system on the surface.[1),2)] The concepts of the sliding surface and the sliding mode are shown in Fig. 1. The nonlinear control applied in the SMC framework is represented by

$$u = u_{eq} - \mathtt{sat}(\sigma(\mathbf{x})/\epsilon)\rho \quad \cdots (1)$$

where u_{eq} is the so called *equivalent control*[1),2)], ρ represents the switching action in the control force, sat is the *saturation function*[1)] and ϵ is a small constant representing a *boundary layer* around the sliding surface and is introduced to eliminate the *chattering effects*[1)] due to control switching.

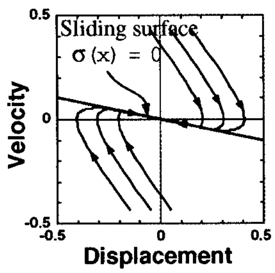

Fig. 1 Sliding surface and the sliding mode.

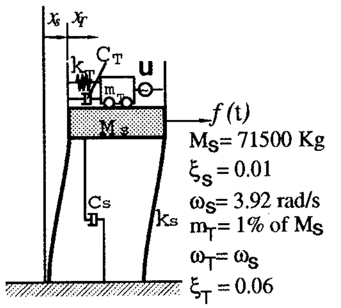

Fig. 2 SDOF model of a tall building with ATMD.

The state–space form of the equation of motion of the ATMD–building model shown in Fig. 2 is written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{H}f(t), \quad \cdots (2)$$

where $\mathbf{x} = [x_s \; x_T \; \dot{x}_s \; \dot{x}_T]'$ is the state–vector, $\mathbf{A}$ is a 4x4 system matrix, $\mathbf{B}$ is the control force location vector of size 4, $\mathbf{H}$ is the external force location vector of size 4 and $f(t)$ is the external force.

Choosing the sliding surface as $\sigma(\mathbf{x}) = \mathbf{S}\mathbf{x}$, where $\mathbf{S}$ is a constant row–vector of size 4, the expression for u_{eq} is obtained by satisfying $\sigma(\mathbf{x}) = 0$, $\dot{\sigma}(\mathbf{x}) = 0$, as

$$u_{eq} = -(\mathbf{SB})^{-1}\left[\mathbf{SAx} + \mathbf{SH}f(t)\right]. \quad \cdots (3)$$

Simulation results of the controlled responses of the structure excited by an inpulse load are shown in Figs. 3 and 4 for $\epsilon = 0.01$, $\rho = 95$ N, $\mathbf{S} = [-5 \;\; 5 \;\; -5 \;\; -1]$ and maximum capacity of the actuator as 2kN. The interaction effect from the ATMD on to the structure is clearly seen in Figs. 3 and 4(a) due to which the structure gets excited instead of being controlled. This is because as both u_{eq} and $\sigma(\mathbf{x})$ are functions of the structure response as well as the TMD response, the large values of the TMD response, Fig. 4(b), results in the large values for u_{eq} which in turn excites the structure. Similarly, the large displacement of TMD, Fig. 4(b), prevents $\sigma(\mathbf{x})$ from approaching to zero and thereby causing a continued nonlinear control action. This continued nonlinear action is undesirable as it may lead to instability of the controlled system.

3. SMC with Modified Sliding Surface for Eliminating the ATMD Interaction

Instead of selecting the sliding surface as a function of the states of the system, let the sliding surface be represented by $\varphi(\mathbf{z}) = \mathcal{S}_1\mathbf{z}_1 + \mathcal{S}_2\mathbf{z}_2$, where $\mathbf{z}$ is realized through the following dynamic system,

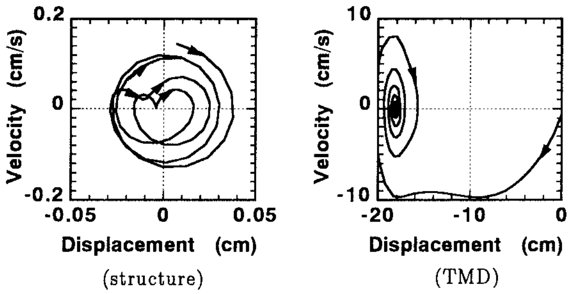

Fig. 3 Phase diagram with conventional SMC.

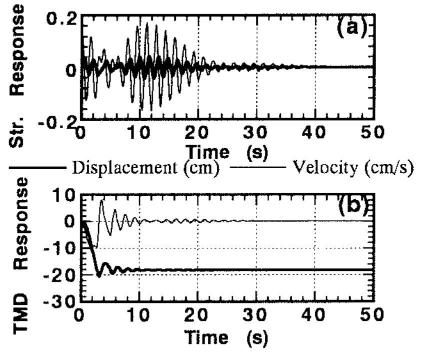

Fig. 4 (a) Structural response, (b) TMD response.

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ L_2 \end{bmatrix} u. \qquad (4)$$

The system of Eq. (4) is essentially a filter which is introduced to impart dynamics to the otherwise non–dynamic sliding surface through a filtering action from $\mathbf{x}$ to $\mathbf{z}$. The elements of matrices $\mathbf{F}, \mathbf{G}$ and $\mathbf{L}$ of Eq. (4) can be determined by forming an augmented system by combining the systems represented by Eq. (2) and Eq. (4) and thereby using LQR theory or pole allocation method. In the present study, the following values were selected by using LQR theory; $F_{11} = -1.2, F_{12} = -1, F_{21} = -1.8, F_{22} = -10, L_1 = 0, L_2 = -1.4, G_1 =$ [0 0 0 0], and $G_2 =$ [−357496.85 696.17 40814.03 1416.05]. $\mathcal{S}_1$ and $\mathcal{S}_2$ were selected to be 1 and 5, respectively.

By satisfying $\varphi(\mathbf{z}) = 0$, $\dot\varphi(\mathbf{z}) = 0$, the equivalent control force for this case is obtained to be

$$u_{eq} = -(\mathcal{S}_2 L_2)^{-1}\left[(\mathcal{S}_1 G_1 + \mathcal{S}_2 G_2)\mathbf{x} + \{\mathcal{S}_1(F_{11} - F_{12}\mathcal{S}_2^{-1}\mathcal{S}_1) + \mathcal{S}_2(F_{21} - F_{22}\mathcal{S}_2^{-1}\mathcal{S}_1)\}z_1\right], \qquad (5)$$

and by selecting a Lyapunov function $V = 0.5\varphi(\mathbf{z})^T\varphi(\mathbf{z})$, and satisfying $\dot V \le 0$, it can be shown that

$$\rho \ge |(\mathcal{S}_2 L_2)^{-1}(\mathcal{S}_1 F_{12} + \mathcal{S}_2 F_{22})(\mathcal{S}_2^{-1}\mathcal{S}_1 z_1 + z_2)|. \qquad (6)$$

The controlled response of the system by applying the control force given by Eq. (1), with u_{eq} and ρ as defined by Eqs. (5) and (6), and $\sigma(\mathbf{x})$ being replaced by $\varphi(\mathbf{z})$, is shown in Figs. 5 and 6. It is clear from Figs. 5 and 6 that the interaction from the ATMD is completely eliminated due to the filtering action of the newly introduced dynamical system represented by Eq. (4).

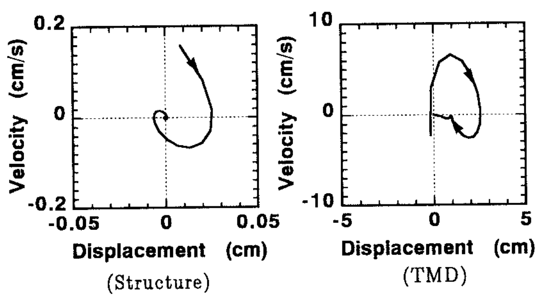

Fig. 5 Phase diagram with modified SMC.

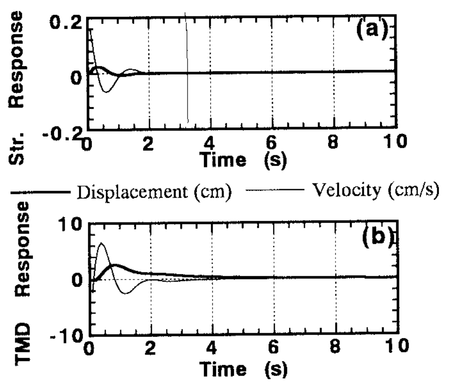

Fig. 6 (a) Structural response, (b) TMD response.

4. Concluding Remarks

It is shown that in SMC, the interaction between the ATMD and the structure can be completely eliminated by injecting dynamics to the otherwise non–dynamic sliding surface by appropriately designing a filter on the sliding surface. It is also worth mentioning that a response–shaping of the controlled response can be achieved with a proper choice of the filter parameters.

References

1) Slotine, J.-J. E., and Li, W."Applied Nonlinear Control". Prentice-Hall International Editions (1991).
2) Utkin, V.I. "Variable structure systems with sliding modes." IEEE Trans. Auto. Cont. **AC–22**(2), 212–222,(1977).