I -430

Linear-Saturation Control of MDOF Structure by Modal Approach

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1. INTRODUCTION

The combination of the saturation(bang-bang type) and linear control is proposed as a new control method for suppressing the responses of the structures under external excitations. The saturation control is used due to its ability to utilize the actuator to its maximum capacity. The contraint on the control force is considered explicitly in the optimization process. On the other hand, when the response is small, the linear control is applied instead of the saturation control in order to avoid the chattering problem. Furthermore, to deal with the MDOF system, the concept of modal control is applied so that only dominant modes are controlled. Finally, the building model of *21-DOF* with AMD control system is simulated to demonstrate the effectiveness of the control system.

2. MODAL CONTROL

At first, the equation of motion is rewritten in the form of state equation as:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{F}(t) \qquad (1)$$

where the state vector $\mathbf{x}(t)=[\mathbf{q}(t)\ \ \dot{\mathbf{q}}(t)]^{\mathrm{T}}$ contains the displacement and velocity vector. $\mathbf{A}$ and $\mathbf{B}$ are the system matrices:

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\end{bmatrix}$$

And $\mathbf{F}(t)=\mathbf{f}u(t)$ is the control force vector where $\mathbf{f}$ is the vector of actuator position and $u(t)$ is the control force exerted by the actuator.

Eq.(1) can be decomposed into the modal state equation by using eigenanalysis and rearranging the equation to have the form

$$\dot{\mathbf{w}}_j(t)=\mathbf{\Lambda}_j\mathbf{w}_j(t)+\mathbf{Z}_j(t) \qquad (2)$$

where the subscript j stands for mode number, $\mathbf{w}_j(t)$ is the modal state vector, and $\mathbf{Z}_j(t)=\mathbf{W}_j u_j(t)$ is the modal control vector. The control force for each mode can be obtained independently. Then the control law in the physical coordinate takes the form

$$u(t)=\frac{1}{2}\mathbf{f}^{\mathrm{T}}\mathbf{M}(\mathbf{V}_{\mathrm{RI}}^{\mathrm{T}})^{*}\mathbf{Z}(t) \qquad (3)$$

where $(\mathbf{V}_{\mathrm{RI}}^{\mathrm{T}})^{*}=(\mathbf{V}_{\mathrm{RI}}\mathbf{V}_{\mathrm{RI}}^{\mathrm{T}})^{-1}\mathbf{V}_{\mathrm{RI}}$ is the pseudo-inverse of $\mathbf{V}_{\mathrm{RI}}^{\mathrm{T}}$ which can be obtained from rearranging the left eigenvector, and $\mathbf{Z}(t)$ is the control force matrix.

$$\mathbf{Z}(t)=[\mathbf{Z}_1(t)\ \mathbf{Z}_2(t)\ \cdots\ \mathbf{Z}_n(t)]^{\mathrm{T}} \qquad (4)$$

3. SATURATION CONTROL

After obtaining Eq.(2), the saturation control law for each mode can be developed seperately. The modal performance index is defined as

$$J_j=\frac{1}{2}\int_0^{\mathrm{T}}\mathbf{w}_j^{\mathrm{T}}(t)\mathbf{Q}_j\mathbf{w}_j(t)\,dt \qquad (5)$$

To minimize the preformance index in Eq.(5) with constraint in Eq.(2). The Hamiltonian is formed by adjoining Eq.(2) to Eq.(5).

$$H_j=\frac{1}{2}\mathbf{w}_j^{\mathrm{T}}(t)\mathbf{Q}_j\mathbf{w}_j(t) -\mathbf{p}_j^{\mathrm{T}}(t)\left(\mathbf{\Lambda}_j\mathbf{w}_j(t)+\mathbf{W}_j u_j(t)\right) \qquad (6)$$

where $\mathbf{p}_j(t)$ is the costate vector. The necessary conditions are

$$\dot{\mathbf{w}}_j(t)=\mathbf{\Lambda}_j\mathbf{w}_j(t)+\mathbf{W}_j u_j(t) \qquad (7)$$

$$\dot{\mathbf{p}}_j(t)=-\mathbf{Q}_j\mathbf{w}_j(t)+\mathbf{\Lambda}_j\mathbf{p}_j(t) \qquad (8)$$

and the control law is

$$u_j(t)=-u_{cj}\mathrm{sgn}\left(\mathbf{W}_j^{\mathrm{T}}\mathbf{p}_j(t)\right) \qquad (9)$$

Since the control force in Eq.(9) is not an explicit function of the state vector, the switching syrface used to determine the control sign must be developed. Start from the origin, Eq.(7) and (8) are integrated backward according to the control law in Eq.(9). Then the switching surface is approximated from the points where the control force changes its sign. However, the control force has the limited capacity which is

$$u(t)\le|u_c| \qquad (10)$$

where u_c is the control capacity. Therefore, control capacity of each mode (u_{cj}) must be specified such that the control force in Eq.(3) does not exceed the limits. This can be done by substituting the control capacity from Eq.(9) and Eq.(10) into Eq.(3) and rewriting the equation in the simple form.

$$u_c=\sum_{j=1}^{n}c_j u_{cj} \qquad (11)$$

where c_j is the coefficient related to the actuator placement, the mass matrix, and the left eigenvector. In this paper, we specify that u_{cj} is proportional to its modal energy.

$$\frac{u_{c1}}{E_1}=\frac{u_{c2}}{E_2}=\ldots=\frac{u_{cn}}{E_n} \qquad (12)$$

Substituting Eq.(12) into Eq.(11), the modal control capacity can be written as

$$u_{cj}=u_c E_j\Big/\sum_{i=1}^{n}c_i E_i \qquad (13)$$

where E_j is the modal energy. According to Eq.(13), the control system will try to control all modes but the effort in control is different depend on the modal energy. Practically, only the few critical modes are needed to be considered as will be shown later.

4. LINEAR CONTROL

The control system will switch to use linear (LQ) control when the vibration energy becomes lesser than the specified energy level. Since the control capacity is limited, we have to specify the penalty on control force(R) such that the required control force is not exceed to limits. From LQ control, we have the performance index:

$$J = \frac{1}{2}\int_0^T \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + Ru^2(t)\,dt \qquad (13)$$

With given **Q** and R , the control gain **G** can be obtained from solving the Riccati's equations. The corresponded boundaries of the region in the state space where this control gain **G** is applicable are

$$\mathbf{G}\mathbf{x}(t) = \pm u_c \qquad (14,15)$$

Eq.(14) and (15) represent two linear hyper surface which are symmetric to the origin. Because boundary of the constant energy level region is nothing but the hyper ellipsoidal. It is the simple optimization problem to find the maximum ellipsoidal which can be fitted in to the regoin between two boundary plane. The solution can be obtain by solving the matrix equation.

$$\mathbf{E}\mathbf{z} + \mathbf{u} = \mathbf{0} \qquad (16)$$

where $\mathbf{z} = [\mathbf{x}\ \lambda]^T, \mathbf{u} = [0 \cdots 0\ u_c]^T$,

$$\mathbf{E} = \begin{bmatrix} \mathbf{N} & \mathbf{G}^T \\ \mathbf{G} & \mathbf{0} \end{bmatrix}, \mathbf{N} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix},$$

and **M**, **K** are the mass and stiffness respectively.

From the state vector(**x**), we can computed the vibration energy. As a result, we can have a relationship between R and the vibration energy.

5. EXAMPLE

The model of *21-DOF* building was used in the simulation with the El-Centro(NS) earthquake loading. The active mass driver(AMD) control system was installed on the top of the building. The control force capacity was set at 200 kN. From the simulation, it was found that only the first and second mode whose properties are shown in the table below contain most of the vibration energy.

No.	period(s)	m(ton)	k(kN/m)
1	5.67	4.05e4	4.97e4
2	2.17	4.24e4	3.54e5

Therefore, only the first and the second modes were considerered. At first, the switching surface of each mode was developed and approximated by the linear line in the modal coordinate. The earthquake intensity was 100 gal. The energy level for linear control was specified at 2.0e5 Joule. During the simulation, the modal state vector and the modal energy were computed. The modal control forces were computed from the modal state vector and switching surface. Then the global control force was computed from Eq.(3). The result is as shown in Figs. 1-3. The control shows good suppression of the response in Fig. 1 which shows the displacement on the top of the building.

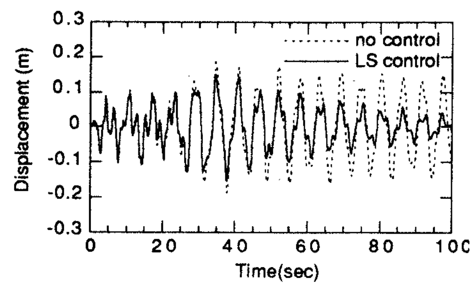

Figure 1 Disp. on the top of building

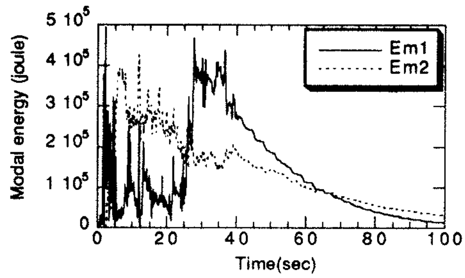

Figure 2 Modal energy

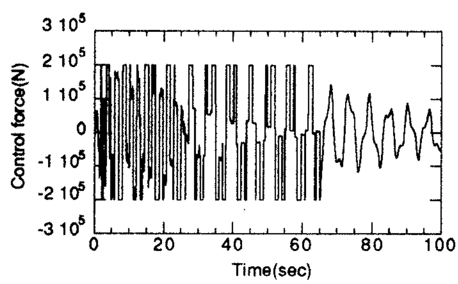

Figure 3 Control force time history

As can be seen from Figs. 2 and 3, the control force tried to suppress the mode with more energy (the 2nd mode for the first 25 sec. And 1st mode for 25-65 sec. And the control finally switched to linear control at low energy level.

6. CONCLUSION

This paper has proposed a new control method which is suitable for application in civil engineering structures because it works explicitly with the constraint on control force. So, the control system is able to operate even the severe loading conditions. The multimode control through modal control was shown to be possible for saturation control. Other aspects such as output feedback control, spillover problem, robustness and stability of the system are to be considered in the future.