I -239 OPTIMAL DESIGN OF PRESTRESSED CONCRETE BEAM SUBJECT TO SERVICEABILITY AND ULTIMATE LIMIT STATE CONSTRAINTS

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1. Introduction

In this paper, an optimum design method for prestressed concrete structures is studied, in which the optimum prestressing force, tendon layout, height and width of uniformed cross section are determined from the economical standpoint. The structure is subjected to the stress constraints in the serviceability limit state, and the flexural-strength design constraints and ductility constraints in ultimate limit state without moment redistribution (elastic design) specified in ACI code. The rigorousness of the method is illustrated by comparing the optimum solutions in which the height of cross section is changed discretely and the height is considered as the design variables.

2. Optimal design problem of prestressed concrete structures

In this study, a uniformed cross section is assumed to be rectangular and the cross section is depicted in Fig.1. From the practical design viewpoint, the layout of tendon is idealized to be parabolic, but the tendon in a member element is assumed to be straight for the calculation of tendon length l_i. As the design variables, prestressing force P and tendon eccentricities e in the middle of spans and interior supports from the center of the cross section, height H and width B of a uniformed cross section in structures are taken into account. The cross-sectional area of tendon A_{ps} is determined by P/f_{pe} where f_{pe} is the permissible tensile stress of prestressing tendon. The primary optimal design problem is formulated as to find the P, e, H and B which minimize the total cost of the structure W subject to the following design constraints of stress limitations in serviceability limit state, and the flexural-strength design criterion and sufficient ductility criterion in ultimate limit state without moment redistribution.

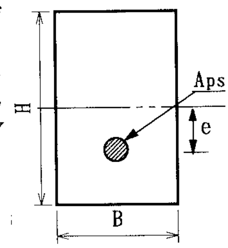

Fig. 1 Cross section

The stress constraints in serviceability limit state are given as

in the case at transfer

for tension $g_{\sigma j} = |\sigma_j| - 3\sqrt{f'_{ci}} \le 0$ (1)

for compression $g_{\sigma j} = |\sigma_j| - 0.6 f'_{ci} \le 0$ (2)

in the case at service

for tension $g_{\sigma j} = |\sigma_j| - 6\sqrt{f'_c} \le 0$ (3)

for compression $g_{\sigma j} = |\sigma_j| - 0.45 f'_c \le 0$ (4)

where $|\sigma_j|$ is the stress at top or bottom fiber in the section j. f'_{ci} and f'_c are, respectively, the concrete strength developed at the time of transfer of prestressing force and the compressive strength of concrete(28 days). At the transfer, the dead loads and initial prestressing force are considered as external loads. At the service, the dead loads, live loads and effective prestressing force are considered.

The flexural-strength and ductility constraints in ultimate limit state are given by

for the flexural-strength design criterion $g_{fj} = M_{uj} - \phi M_{nj} \le 0$ (5)

for the sufficient ductility criterion $g_{dj} = \omega_{pj} - 0.24\beta_1 \le 0$ (6)

where $M_u(=1.4M_D+1.7M_L)$ and $M_n(=A_{ps}f_{ps}(d_p-0.5a))$ are, respectively, the factored moment at the section and the nominal moment capacity of section. ϕ is the flexural strength reduction factor. ω_p $(=A_{ps}f_{ps}/bd_p f'_c)$ and $\beta_1(=a/c)$ are the prestressing reinforcement index and the ratio of the depth of equivalent rectangular stress block (a) to the distance from the maximum compressive fiber to the neutral axis (c), respectively. M_D and M_L are the bending moments due to dead loads and live loads. f_{ps} is stress in prestressed reinforcement at nominal strength at the section. b and d_p express the width of compressive area in a cross section and the distance from centroid of prestressed reinforcement to extreme compression fiber at the section, respectively.

The maximum and minimum bending moments due to live loads are calculated by applying a uniformly distributed live load to each span and summing up all positive or negative bending moment separately. The secondary bending moment due to prestressing force is obtained by considering the primary prestressing bending moment as the equivalent loads at each element. In the analysis of structures, the section properties such as cross-sectional area and moment of inertia are calculated by taking the mean values of the properties at both-end nodes of each member element.

3. Optimum design algorithm

Utilizing the convex and linear approximation concept, the primary optimal design problem can be approximated as the following convex and separable subproblem by using the first-order partial derivatives with respect to the design variables and the direct and reciprocal design variables.

Find P, e, H, B which

$$\text{minimize} \quad \Delta W(P,\mathbf{e},H,B)=\sum_{i=1}^{m}(\omega_{pi}P+\sum_{k=1}^{n}\omega_{eki}e_k+\omega_{Hi}H+\omega_{Bi}B) \tag{7}$$

$$\text{subject to} \quad g_j(P,\mathbf{e},H,B)=a_{j_{(+)}}P-a_{j_{(-)}}(P^0)^2\frac{1}{P}+\sum_{k=1}^{n}[y_{jk_{(+)}}e_k-y_{jk_{(-)}}(e_k)^0\frac{1}{e_k}]+h_{j_{(+)}}H-h_{j_{(-)}}(H^0)_2\frac{1}{H}$$

$$+b_{j_{(+)}}B-b_{j_{(-)}}(B^0)^2\frac{1}{B}+\overline{U}_j\le 0 \qquad (1,\ldots,q) \tag{8}$$

$$P^l\le P\le P^u,\quad e_k^{\,l}\le e_k\le e_k^{\,u}\ (k=1,\ldots,n),\quad H^l\le H\le H^u,\quad B^l\le B\le B^u \tag{9}$$

$$\text{where} \quad \overline{U}_j=g_j(P^0,\mathbf{e}^0,H^0,B^0)-P^0[a_{j_{(+)}}-a_{j_{(-)}}]-\sum_{k=1}^{n}e^0_k[y_{jk_{(+)}}-y_{jk_{(-)}}]-H^0[h_{j_{(+)}}-h_{j_{(-)}}]-B^0[b_{j_{(+)}}-b_{j_{(-)}}]$$

$$\omega_p=\frac{\partial W}{\partial P},\ \omega_{ek}=\frac{\partial W}{\partial e_k}\ \omega_H=\frac{\partial W}{\partial H},\ \omega_B=\frac{\partial W}{\partial B},\ a_j=\frac{\partial g_j}{\partial P},\ y_{jk}=\frac{\partial g_j}{\partial e_k},\ h_j=\frac{\partial g_j}{\partial H},\ b_j=\frac{\partial g_j}{\partial B}$$

m is the number of elements. The sensitivities of W and g_j with respect to P, $\mathbf{e}$, H and B are calculated by using the forward difference method.

The above approximated subproblem is solved by a dual method where the separable Lagrangian function is minimized with respect to the design variables and maximized with respect to Lagrange multipliers (dual variables). At the minimization process, the design variables are improved by simple expressions derived from stationary conditions of separable Lagrangian function. Then at the maximization process, the dual variables are improved by a Newton-type algorithm. The optimum solution is obtained by iterating the above min.-max. process.

4. Numerical design examples and discussions

The above method has been applied to various minimum cost designs of prestressed concrete structures. In this paper, the numerical results for the three-span prestressed concrete continuous beam shown in Fig. 2 are discussed.

In the numerical design example, f'_c, f'_{ci}, f_{pe} and f_{ps} are, respectively, set at 41 Mpa, 31 Mpa, 1,665 Mpa and 1,103 Mpa. β_1 and ϕ are 0.85 and 0.9. The unit costs of prestressing tendon and concrete are 6916800 /m^3 and 24000 /m^3, respectively. The structure is divided into 24 member elements in order to obtain the accurate results.

Fig. 3 shows the comparison of two optimum solutions, one for P and $\mathbf{e}$ design variables (case A) and another for P, $\mathbf{e}$, H and B design variables (case B). For the case A, the optimum solution is obtained after 8 iterations. The results are drawn in the dotted line. For the case B, the optimum solution is obtained after 18 iterations. In this case, the width B is determined by the lower limit (200 mm). As clearly seen from Fig. 3, the shape of the tendon is just proportional to the distribution of the bending moments throughout the beam, then the optimum solution seems to be quite reasonable. The total cost for case B (504209) is 42% less than that for case A (874042). The quite similar optimum solutions are obtained from various starting points. Therefore, it can be said that the proposed optimum design algorithm is quite reliable.

To illustrate the rigorousness of the method, the effect of the height of cross section H on optimum solution is investigated by changing H discretely. Fig. 4 shows the variation of total cost W, P and e_3 due to the change of H where B is fixed at 200 mm. When H is lower than 1.0 m, optimum solutions can not be obtained by improvement of P and $\mathbf{e}$. By the increment of H, P is decreased and e_3 is increased, but the tendency of variations of P and e_3 due to the change of H is different to the combinations of P and $\mathbf{e}$. However, total cost W is linearly increased by the increment of H. Fig. 4 shows that the optimum H is nearly 1.0 m. The optimum H and total cost is quite similar to the optimum solution for case B.

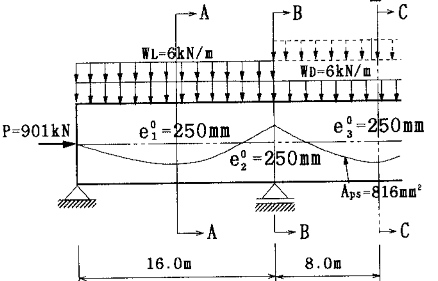

Fig.2 Initial 3-span continuous prestressed concrete beam

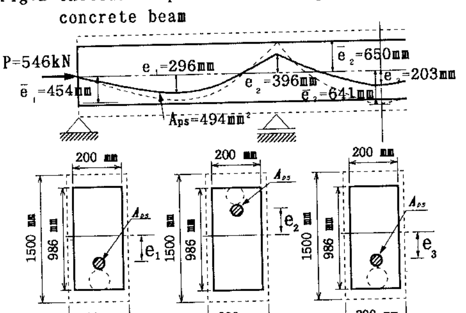

Fig.3 Comparison of the optimum solutions
------ : Opt. Solution for P,e design variables (A)
——— : Opt. Solution for P,e,H,B design variables (B)

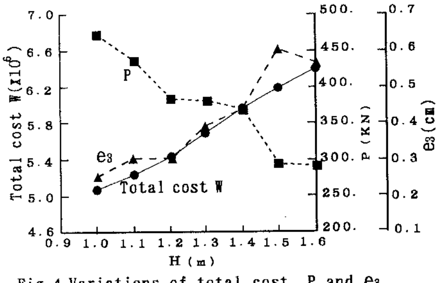

Fig.4 Variations of total cost, P and e3 due to the change of H