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A Kinked Crack Initiated by an Inclined Circular Punch with One End Sliding and the Other End with a Sharp Corner on a Half Plane

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1. Introduction A flat-ended punch problem with a vertical crack on the surface of a half plane was researched in the previous papers [1,2], and a circular punch problem with an oblique crack has been considered recently[3]. In the present paper, a circular inclined punch problem with one end sliding and the other end with a sharp corner is considered to initiate a kinked surface crack on the half plane. The half plane with a kinked crack is mapped into a unit circle by a rational mapping function so that the solution of the problem can be obtained explicitly by solving a Riemann-Hilbert equation. Since the stress components at the sliding end of the punch are finite, the property can be used to decide the length of the contact region; on the other hand, the resultant moment about the center of the punch must vanish if the load is acted at the center of the punch. The fact can be employed to determine the inclined angle of the punch. The stress intensity factors of the crack are calculated with different frictional coefficients and kinked lengths.

2. The mapping function The half plane with a surface kinked crack is mapped into a unit circle by the following rational mapping function:

$$z = \omega(\zeta) = \frac{E_0}{1-\zeta} + \sum_{k=1}^{36+n} \frac{E_k}{\zeta_k - \zeta} + E_c \tag{1}$$

where E_0, E_k and ζ_k are known coefficients, which are changed with the inclined angle of the original and kinked part of the crack as well as their lengths; E_c is related to the distance from the crack to the punch, and n is usually selected within 16-24 with enough precision. It is not stated how to form (1) owing to the limited length of the paper.

3. The loading conditions As shown in Fig.1, the loading conditions of the problem can be expressed as

$$\begin{aligned} &p_x = p_y = 0 && on\ L = L_1 + L_2 \\ &p_x = \mu p_y,\ \int p_y ds = P && on\ M \\ &V = -\varepsilon x + x^2/2R && on\ M \end{aligned} \tag{2}$$

where $L_1 = ABCDEFG$, $L_2 = HA$, $M = GH$, ε is the inclined angle of the punch, which is taken as a positive value when the punch is inclined in a clockwise direction; R is the radius of the curvature of the punch, and μ is the Coulomb's frictional coefficient on M.

4. The complex stress functions In terms of (2), the problem can be transformed into a Riemann-Hilbert equation. By solving the R-H equation, one of the stress functions can be expressed as

$$\phi(\zeta) = H_A(\zeta) + H_B(\zeta) + H_C(\zeta) + \frac{1+i\mu}{2} J(\zeta) + Q(\zeta)\chi(\zeta) \tag{3}$$

where $H_A(\zeta)$, $H_B(\zeta)$ and $H_C(\zeta)$ are related to P, R and ε, respectively [3].

$$Q(\zeta)\chi(\zeta) = -\sum_{k=1}^{36+n} \frac{\chi(\zeta)\overline{A_k}B_k}{\chi(\zeta_k)(\zeta_k - \zeta)},\ J(\zeta) = -\sum_{k=1}^{36+n}\left[1 - \frac{\chi(\zeta)}{\chi(\zeta_k)}\right]\frac{\overline{A_k}B_k}{\zeta_k - \zeta} + \frac{1-i\mu}{1+i\mu}\sum_{k=1}^{36+n}\left[1 - \frac{\chi(\zeta)}{\chi(\zeta'_k)}\right]\frac{A_k \overline{B_k}\zeta'^2_k}{\zeta'_k - \zeta}$$

and $A_k = \phi'(\zeta'_k)$, $B_k = E_k/\overline{\omega'(\zeta'_k)}$, $\zeta'_k = 1/\overline{\zeta_k}$, $\chi(\zeta) = (\zeta-\alpha)^m(\zeta-\beta)^{1-m}$, $m = 0.5 - i\ln g/2\pi$

$g = [(\kappa+1) - i\mu(\kappa-1)]/[(\kappa+1) + i\mu(\kappa-1)]$; $\kappa = 3-4\nu$ for plane strain state and $(3-\nu)/(1+\nu)$ for plane stress state. G and ν are shear modulus and Poisson's ratio of the half plane, respectively.

The other stress function is given by

$$\psi(\zeta) = -\bar{\phi}(1/\zeta) - \bar{\omega}(1/\zeta)\phi'(\zeta)/\omega'(\zeta) \tag{4}$$

5. The length of the contact region The stress components on the boundary of the half plane can be expressed as

$$\sigma_{rr} + i\sigma_{r\theta} = \frac{1}{\omega'(\sigma)}\left\{\frac{1-i\mu}{2}\frac{i}{\pi}\frac{(1-\alpha)(1-\beta)}{\chi(1)(1-\sigma)(\sigma-\alpha)(\sigma-\beta)} + \frac{f(\sigma)}{\sigma-\alpha} + \frac{g(\sigma)}{\sigma-\beta} + h(\sigma)\right\}[\chi^+(\sigma) - \chi^-(\sigma)] \tag{5}$$

where $f(\sigma)$, $g(\sigma)$ and $h(\sigma)$ are known functions.

The following condition can be formed from (5) to satisfy that the stress components at the sliding end are finite:

$$\frac{1-i\mu}{2}\frac{i}{\pi}\frac{1-\alpha}{\chi(1)(\beta-\alpha)}P + g(\beta) = 0 \tag{6}$$

β can then be obtained from (6), and the length of the contact region is decided by

$$a' = \omega(\alpha) - \omega(\beta) \tag{7}$$

The right end of the contact region is assumed to be fixed, while the position of the left end of the contact region changes with other conditions. Fig.2 shows that the length of the contact region increases with the increase of the frictional coefficient and decreases with the increase of the kinked length.

6. The inclined angle of the punch Since the load is acted at the center of the punch, the resultant moment on the contact region must vanish with respect to the center of the punch in the natural state , which can be used as the condition to determine the inclined angle of the punch. With the increase of the frictional coefficient, the inclined angle of the punch decreases. and the larger the kinked length of the crack becomes, the larger the inclined angle becomes, as shown in Fig.3.

7. The stress intensity factors of the crack The stress intensity factors at the tip of the kinked crack are defined as

$$K_I - iK_{II} = 2\sqrt{\pi} e^{-\frac{\delta}{2}i} \frac{\phi'(\zeta_0)}{\sqrt{\omega''(\zeta_0)}} \tag{8}$$

where $\zeta_0 = -1$ is ζ on the unit circle corresponding to the tip of the crack, and $\delta = -(\alpha_0 + \beta_0 - 1)\pi / 180$ represents the angle between the kinked part of the crack and the x-axis.

The non-dimensional stress intensity factors of the crack are defined as

$$F_I^a + iF_{II}^a = \frac{\sqrt{a}(K_I + iK_{II})}{P\sqrt{\pi}} \qquad 0 < b/a \le 1 \tag{9a}$$

$$F_I^l + iF_{II}^l = \frac{\sqrt{b}(K_I + iK_{II})}{P\sqrt{\pi}} \qquad 0 < a/b \le 1 \tag{9b}$$

The stress intensity factors of the crack increase with the increase of the frictional coefficient, and with the increase of the kinked length, F_I decreases, while for F_{II} , it decreases for relative larger frictional coefficients, as shown in Fig.4.

In the above calculation, $\kappa = 2$, $\alpha_0 = 90^0$, $\beta_0 = 120^0$, b/a=0.5, c/a=0.0 and $Ga^2 / PR = 1$ have been selected.

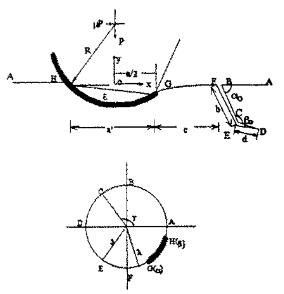

Fig.1 The punch with a kinked crack

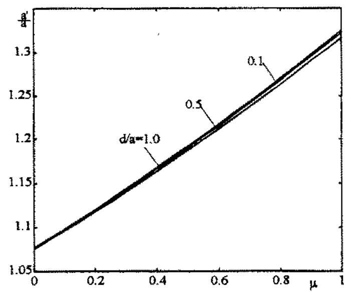

Fig.2 The length of the contact region

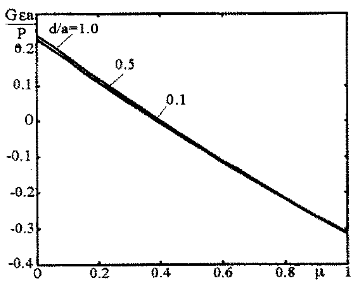

Fig.3 The inclined angle of the punch

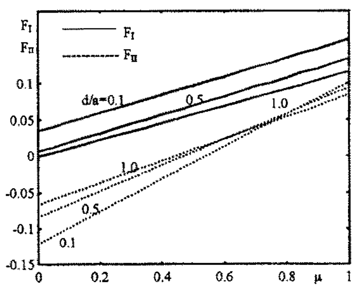

Fig.4 The stress intensity factors of the crack

References

[1]Hasebe. N., Okumura, M. and Nakamura, T., J. Engrg. Mech., ASCE 115-6, 1137-1149(1989)
[2]Okumura. M. , Hasebe , N. and Nakamura, T., J. Engrg. Mech., ASCE 116-10, 2173-2185(1990)
[3]Qian. J.. Hasebe, N. and Nakamura,T.,The 44th Nat.Cong. of Theo. & Applied Mech.,357-358(1995)