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Axially symmetric thermal stress of a penny-shaped crack under general heat flux

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Introduction

A method has been developed to solve a problem of finding distribution of stress in the neighbourhood of a penny-shaped crack in an infinite isotropic elastic solid under general mechanical loading in [1]. Extending the method developed in [1] to thermoelasticity, thermal stress around the penny-shaped crack subjected to general surface temperature have been calculated in [2]. Using the general results of Ref.[1,2] in this paper, the problem of penny-shaped crack whose surfaces are subjected to uneven heat flux (unsymmetric about the crack plane) is solved. In terms of prescribed flux functions, expressions for the mode-I and mode-II thermal stress intensity factors K_I and K_{II} are obtained. In the special case of heat flux functions, stresses at a general point of the medium are obtained and presented graphically.

Solution of the Problem

Let a penny-shaped crack be located in the plane z = 0 of an infinite, homogeneous and isotropic elastic solid . In terms of the cylindrical coordinates (r, ϕ, z) the crack occupies the region $0 \le r \le a \ (z = 0)$. The crack is subjected to axisymmetric surface tractions and heat flux which are not necessarily self-equilibrating. The heat flux applied to the upper surface of the crack ($z \to 0+, 0 < r < a$) is different from the lower surface of the crack ($z \to 0-, 0 < r < a$). Moreover, stress, displacement, temperature and flux functions are continuous outside the crack region r > a z = 0). The stress, displacement and temperature fields at a general point of the medium have been derived in terms of jumps of stress, displacement, temperature and heat flux at the crack plane z=0 (r>0) given by

$$\int_\rho^\infty \frac{r\left[\theta^{(1)}(r,0)-\theta^{(2)}(r,0)\right]dr}{\sqrt{r^2-\rho^2}} = E(\rho),(\rho>0) \quad (1)$$

$$\int_\rho^\infty \frac{r\left[\frac{d}{dz}\theta^{(1)}(r,0)-\frac{d}{dz}\theta^{(2)}(r,0)\right]dr}{\sqrt{r^2-\rho^2}} = F(\rho),(\rho>0) \quad (2)$$

together with (2.7)-(2.10) of Ref.[1] (see Ref.[2]), where superscripts 1 and 2 denote temperature fields for z>0 and z<0 respectively. The jumps A, B, C, D, E, F [1] are derived satisfying the axisymmetric boundary conditions of the crack on the plane z=0. Boundary conditions are

$$\theta^{(1)}(r,0) = \theta^{(2)}(r,0),(r>a) \quad (3)$$

$$\frac{\partial}{\partial z}\theta^{(1)}(r,0) = \frac{\partial}{\partial z}\theta^{(2)}(r,0),(r>a) \quad (4)$$

$$\frac{\partial}{\partial z}\theta^{(1)}(r,0) - \frac{\partial}{\partial z}\theta^{(2)}(r,0) = F_1^*(r),(0<r<a) \quad (5)$$

$$\frac{\partial}{\partial z}\theta^{(1)}(r,0) + \frac{\partial}{\partial z}\theta^{(2)}(r,0) = F_2^*(r),\ (0<r<a) \quad (6)$$

together with equations (3.1)-(3.6) of Ref[1]. At the rim of the crack, displacements and temperature are continuous and radial component of the displacement is zero at origin. In the absence of mechanical load, limiting values of stress, displacements, temperature and heat flux functions as $z \to 0+$ and as $z \to 0-$ and boundary conditions lead to a Abel integral equations. Hence the unknown functions A, B, C, D, E, F are obtained in terms of prescribed quantities.

Substituting A, B, C, D, E, F we can simplify stress and displacement components on the crack plane z=0 and hence we can find the mode-I and mode-II thermal stress intensity factors K_I and K_{II} at the rim of the crack which are given by

$$K_I = \frac{\mu(1+\nu)\alpha}{2(1-\nu)\sqrt{a}}\int_0^a sF_1^*(s)ds \quad (7)$$

$$K_{II} = -\frac{\mu(1+\nu)\alpha}{\pi(1-\nu)a^{3/2}}\int_0^a s\sqrt{a^2-s^2}F_2^*(s)ds \quad (8)$$

For a special case of heat flux, that is $F_1^*(r) = \varepsilon_1$; $F_2^*(r) = \varepsilon_2$, $(0<r<a)$ where ε_1 and ε_2 are constants, stress at a general point is derived. The normal and the shear stress components are given by

$$\sigma_z(r,z) = \frac{\mu(1+\nu)\alpha\varepsilon_1 a}{4(1-\nu)}\mathrm{Re}\left[\frac{a}{\sqrt{r^2+s^2}}+\frac{asz}{\left(r^2+s^2\right)^{3/2}}\right] - \frac{\mu(1+\nu)\alpha\varepsilon_2 a}{3\pi(1-\nu)}\mathrm{Re}\left[\frac{isza}{\left(r^2+s^2\right)^{3/2}}\right] \quad ,z>0 \quad (9)$$

$$\sigma_{rz}(r,z) = \frac{\mu(1+\nu)\alpha\varepsilon_1 a}{4(1-\nu)}\mathrm{Re}\left[\frac{zar}{\left(r^2+s^2\right)^{3/2}}\right]$$

$$+\frac{\mu(1+\nu)\alpha\varepsilon_2 a}{3\pi(1-\nu)}\mathrm{Re}\left[\frac{isa}{r\sqrt{r^2+s^2}}+\frac{zrai}{\left(r^2+s^2\right)^{3/2}}\right], z>0 \quad (10)$$

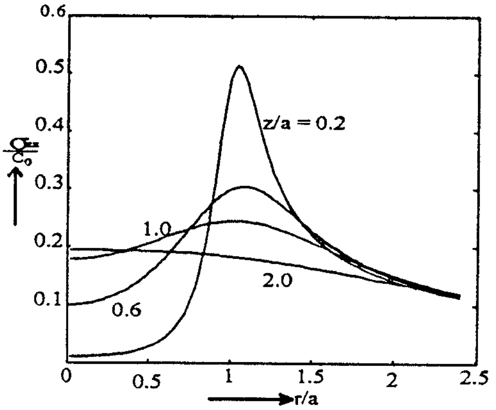

Fig.1 Variation of the stress component σ_{zz}/C_0 with r/a for z>0

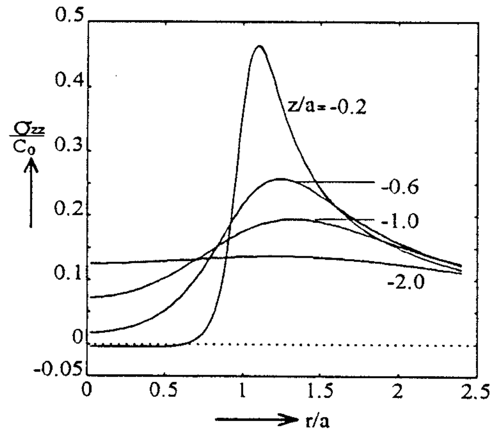

Fig.2 Variation of the stress component σ_{zz}/C_0 with r/a for z <0

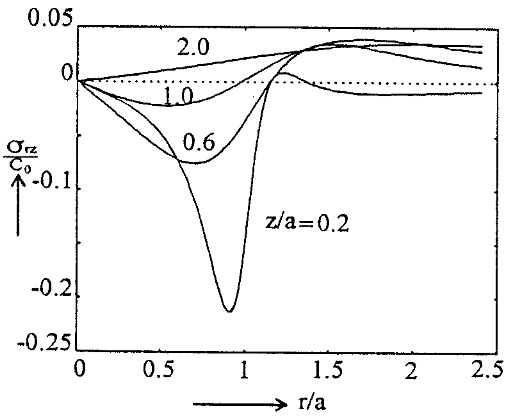

Fig.3 Variation of the stress component σ_{rz}/C_0 with r/a for z >0

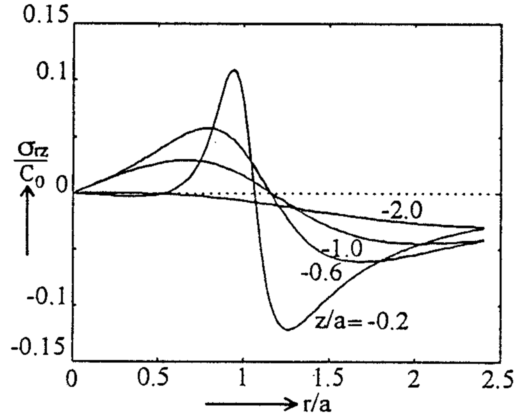

Fig.4 Variation of the stress component σ_{rz}/C_0 with r/a for z <0

where s = z + ia. For z<0 stresses can be obtained using the fact that first [ ] and second [ ] respectively of eqns. (9) and (10) are even functions of z while second and first terms of eqns. (9) and (10) are odd functions of z. μ is modulus of rigidity and ν, α are Poisson ratio and coefficient of linear thermal expansion of the solid, respectively.

Results and Discussion

When $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 0$, the expressions for stress components are same as those given by Olesiak and Sneddon [3].The non-dimensionalised stress components σ_{zz}/C_0, σ_{rz}/C_0 where $C_0 = -\mu(1+\nu)\alpha\varepsilon_0/(1-\nu)$ has been presented in Fig.1-4 for a special case in which $\varepsilon_1 = \varepsilon_0$; $\varepsilon_2 = \varepsilon_0$, $(0 < r < a)$ that is, upper surface of the penny-shaped crack is subjected to constant heat flux ε_0 while the lower surface is kept at the zero flux. The trends of the curves are similar to temperature problem [2] but magnitude of these quantities is less in the present case.

Concluding Remarks

(1) The problem of penny shaped crack under heat flux is more difficult compared to the temperature problem. But the heat flux problem is physically advantageous compared to temperature problem

(2)If the penny shaped crack is subjected to unsymmetric heat flux conditions, both mode-I and mode-II stress intensity factors exist and they are dependent on material constants..

References

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[3]Olesiak, Z., and Sneddon I. N. : Arch. Rational, Mech. Anal., 4 (1959) 238-254.