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# STUDY OF $M-\theta_r$ BEHAVIOR OF UNSTIFFENED END-PLATE CONNECTIONS

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#### 1. INTRODUCTION

For evaluating moment-rotation characteristics of unstiffened extended end-plate connections, a typical of which is shown in Fig. 1, the derivations of ultimate moment capacity and initial connection stiffness are of paramount importance. In this study, from a thorough review, a simplified methodology has been presented.

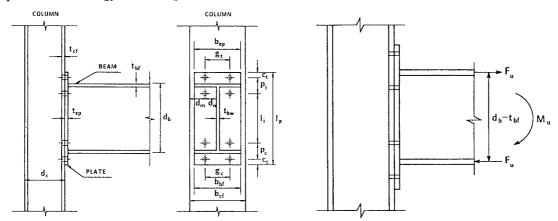


Fig. 1 An unstiffened extended end-plate connection

Fig. 2 Ultimate moment capacity

#### 2. ULTIMATE MOMENT CAPACITY

The moment applied to a beam-to-column connection can be calculated from the idealization that an internal couple consisting of two beam flange forces  $F_u$  with a moment arm of  $(d_b - t_{bf})$  equals the external moment  $M_u$  (Fig. 2), where  $d_b$  is beam depth and  $t_{bf}$  is beam flange thickness. The minimum flange force among the three common failure modes viz: (1) bolt failure, (2) column flange failure, and (3) end-plate failure, will obviously be the governing one for the moment capacity determination. Thus, the ultimate moment capacity will be,

$$M_{u} = F_{u}(\min.) \times (d_{b} - t_{bf})$$
 (1)

#### 2.1. BOLT FAILURE

Flange force for bolt failure can be determined from the bolt force  $F_{bo}$  subtracting the prying force q. Prying force is the bolt tension increment due to end-plate deformation (Fig. 4) which is obtained from Chasten, C.P. et al. (1992). Bolt force is obtained from stress equality assumption.

 $F_{u} = F_{bo} - q = 4A_{bo}\sigma_{yb} - \frac{b_{ep}t_{ep}^{2}\sigma_{yep}}{2.4c_{t}}$  (2)

where  $A_{bo}$  is the cross-sectional area of the bolt shaft,  $\sigma_{yb}$ ,  $\sigma_{yep}$  are the yield stresses of the bolt and end-plate material respectively, and  $b_{ep}$ ,  $t_{ep}$  and  $c_t$  are defined in Figs. 1.

#### 2.2. END-PLATE FAILURE

Surtees and Mann (1970) employed a linear yield line failure pattern and derived the following equation for flange force responsible for the end-plate failure.

$$F_{u} = \sigma_{yep} t_{ep}^{2} \left[ \frac{2b_{ep}}{(p_{t} - t_{bf})} + \frac{1.2(d_{b} - t_{bf})}{(g_{t} - t_{bw})} \right]$$
(3)

where d<sub>b</sub> is the bolt diameter, and p<sub>t</sub>, t<sub>bf</sub>, g<sub>t</sub> and t<sub>bw</sub> are defined in Figs. 1.

### 2.3. COLUMN FLANGE FAILURE

Two mechanisms for unstiffened column flange failure are proposeded by Packer, J.A. et al. (1977): (1) Mechanism A: simultaneous yielding of column flange and bolts (Eq. 4), (2) Mechanism B: yielding of column flange (Eq. 5).

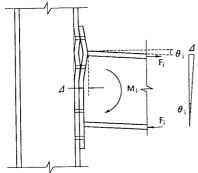
$$F_{u} = \sigma_{ycf} t_{cf}^{2} \left\{ \pi + \frac{0.5p_{t}}{(d_{m} + d_{n})} \right\} + \frac{4A_{bo}\sigma_{yb}d_{n}}{(d_{m} + d_{n})}$$
 (4), 
$$F_{u} = \sigma_{ycf} t_{cf}^{2} \left\{ \pi + \frac{2d_{n} + p_{t} - d_{ho}}{d_{m}} \right\}$$
 (5)

## 3. THE INITIAL STIFFNESS

For calculation of initial stiffness, the principle followed by Yee and Melchers (1986) will be applied here with some modifications. The initial stiffness (Fig. 3) can be expressed as:

$$R_{ki} = \frac{M_i}{\theta_i} = \frac{F_i (d_b - t_{bf})}{\theta_i} = \frac{F_i (d_b - t_{bf})^2}{\Delta} = \frac{F_i (d_b - t_{bf})^2}{\Delta_{ep} + \Delta_{cf}}$$
(6)

in which  $M_i$  is initial moment,  $F_i$  is flange force,  $\theta_i$  is initial rotation,  $\Delta$  = total deformation occurred at the connection,  $\Delta_{ep}$  = deformation due end-plate flexure and  $\Delta_{cf}$  = deformation due column flange flexure.



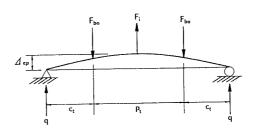


Fig. 3 Connection deformation

Fig. 4 Simply supported model for end-plate flexure

The  $\Delta_{\rm ep}$  and  $\Delta_{\rm cf}$  are obtained from a simply supported T-stub flexural idealization (Fig. 4), in which, the flange force  $F_{\rm i}$  is being resisted by the bolt forces  $F_{\rm bo}$  and the prying forces q become the reactions at the supports. The  $\Delta_{\rm ep}$  and  $\Delta_{\rm cf}$  are then determined from simple bending theory:

$$\Delta_{\rm ep} = \frac{F_{\rm i} Z_{\rm ep}}{E} \left[ \frac{1}{8} - \frac{q_{\rm s}}{2} \left( \frac{3}{4} \alpha_{\rm ep} - \alpha_{\rm ep}^{3} \right) \right] (7), \quad \Delta_{\rm cf} = \frac{F_{\rm i} Z_{\rm cf}}{E} \left[ \frac{1}{8} - \frac{q_{\rm s}}{2} \left( \frac{3}{4} \alpha_{\rm cf} - \alpha_{\rm cf}^{3} \right) \right] (8)$$

where

$$q_{s} = \frac{Z_{ep} (1.5\alpha_{ep} - 2\alpha_{ep}^{3}) + Z_{cf} (1.5\alpha_{cf} - 2\alpha_{cf}^{3})}{Z_{ep} (6\alpha_{ep}^{2} - 8\alpha_{ep}^{3}) + Z_{cf} (6\alpha_{cf}^{2} - 8\alpha_{cf}^{3}) + 1.25 (t_{ep} + t_{cf})/A_{bo}}$$
(9)

and

$$Z_{ep} = \frac{2 \left(2 c_{t} + p_{t}\right)^{3}}{b_{ep} \ t_{ep}^{3}} \ (10), \quad Z_{cf} = \frac{2 b_{cf}^{3}}{\left(2 c_{t} + p_{t}\right) \ t_{cf}^{3}} \ (11), \quad \alpha_{ep} = \frac{c_{t}}{2 c_{t} + p_{t}} \ (12), \quad \alpha_{cf} = \frac{b_{cf} - gt}{2 b_{cf}} \ (13)$$

## 4. CONCLUSION

A simplied analytical approach to calculate ultimate moment capacity  $M_u$  and initial stiffness of unstiffened extended end-plate connections  $R_{ki}$  is proposed. Comparison with some experimental investigations revealed the justification of the analytical approach.

## 5. REFERENCES

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