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STABILITY DESIGN PROCEDURE OF PLATE GIRDERS WITH UPPER FLANGE RESTRAINED BY CONCRETE SLAB

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1. INTRODUCTION

I-section members are used in plate girders in a variety of structures where the upper flange is either embedded in a reinforced concrete deck slab or connected by studs to the deck slab. The usual procedure to analyze such members is to adopt an equivalent cross-section of the girder composed of the steel I-section and an equivalent steel plate with thickness equal to the reinforced concrete deck slab thickness divided by n which is defined as E_s / E_c where E_s and E_c are the modulus of elasticity of steel and concrete, respectively. The present code [1] recommends the use of a constant value of $n = 7$ in spite of the fact that the modulus of elasticity of concrete E_c depends on the strength of concrete. In this paper a proposal is presented to analyze plate girders as I-section members with continuously distributed elastic springs. The actual value of the modulus of elasticity of concrete will be incorporated in estimating the elastic stiffness constants of the springs and hence, the use of the ratio n will not be necessary.

2. EQUILIBRIUM OF I-SECTION BEAM-COLUMN EMBEDDED IN ELASTIC MEDIA

Consider an I-section with single axis of symmetry, Y-axis, as shown in Fig. 1 where OX and OY are the principal centroidal axes. The section is considered to be elastically restrained at point H from being displaced in the plane OXY. The elastic stiffness constants representing the elastic lateral support are K_ξ, K_η and K_θ where ξ and η represent the displacements in X and Y directions, respectively, and θ represents the torsional angle. Considering the lateral external loads in X and Y directions as q_x and q_y, respectively, and the torsional moment as m, The differential equilibrium equations representing this case have been derived by Vlasov [2] as follows

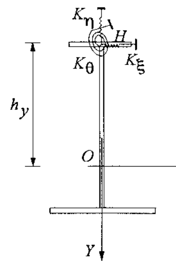

Fig. 1. I-Section with elastic supports

$$
\begin{aligned}
&EI_y\xi^{IV}+K_\xi[\xi-(h_y-a_y)\,\theta]=q_x\\
&EI_x\eta^{IV}+K_\eta\,\eta=q_y\\
&EI_w\theta^{IV}-GJ\theta^{II}+[K_\xi(h_y-a_y)^2+K_\theta]\,\theta-K_\xi(h_y-a_y)\,\xi=m
\end{aligned}
\tag{1}
$$

3. STABILITY OF BEAM-COLUMN EMBEDDED IN ELASTIC MEDIA

Consider a simply supported beam-column of length L and a cross-sectional area A with single axis of symmetry, Y-axis, subject to axial compressive forces P, applied at the centroid, at both ends together with equal and opposite external moments M_x. The functions of the external loads q_x, q_y and m of Eq. (1) are as follows

$$
\begin{aligned}
&q_x=-P\xi^{II}-(a_yP+M_x)\,\theta^{II} \qquad \& \qquad q_y=-P\eta^{II}\\
&m=-(a_yP+M_x)\,\xi^{II}-(Pr^2-2\beta_yM_x)\,\theta^{II} \qquad \left(\beta_y=\frac{1}{2I_x}\int_A y(x^2+y^2)dA-a_y\right)
\end{aligned}
\tag{2}
$$

where r is the radius of gyration of the cross-section. The displacement functions satisfying the simply-supported boundary conditions with restricted torsional rotation and free warping at both ends are as follows

$$\xi=C_1\sin\lambda z\ ,\ \eta=C_2\sin\lambda z\ ,\ \theta=C_3\sin\lambda z \qquad (\lambda=n\pi/L\ ,\ n=1,2,3,\dots) \tag{3}$$

where C_1, C_2 and C_3 are arbitrary constants. Differentiating Eq. (3) with respect to z and substituting, together with Eq. (2), into the differential equilibrium equations, Eq. (1), and dividing by the common factor ($\sin\lambda z$), the following homogenous equations could be obtained

$$
\begin{aligned}
&C_1[EI_y\lambda^4-P\lambda^2+K_\xi]-C_3[(a_yP+M_x)\,\lambda^2+K_\xi(h_y-a_y)]=0\\
&C_2[EI_x\lambda^4-P\lambda^2+K_\eta]=0\\
&-C_1[(a_yP+M_x)\,\lambda^2+K_\xi(h_y-a_y)]+C_3[EI_w\lambda^4+(Pr^2-2\beta_yM_x-GJ)\lambda^2+K_\xi(h_y-a_y)^2+K_\theta]=0
\end{aligned}
\tag{4}
$$

The arbitrary constants C_1, C_2 and C_3 cannot be equal to zero. Therefore, to have a solution for Eq. (4) other than the trivial solution, the value of the determinant of coefficients should be equal to zero. The values of P and M_x satisfying this condition are to be the critical values for the axial compressive force and bending moment to cause instability to the beam-column.

4. IN-PLANE INSTABILITY OF A SIMPLY SUPPORTED PLATE GIRDER

In case of a simply-supported plate girder where the upper flange is embedded in a reinforced concrete deck slab, a proposal is presented to represent the concrete deck slab as uniformly distributed elastic springs with elastic stiffness constants depending on the actual value of E_c.

The elastic spring constant representing the concrete deck slab can be obtained by equating the displacement of a simply-supported beam of L and flexural rigidity equal to E_cI_c with the displacement of an elastic beam rested on uniformly distributed elastic springs of elastic stiffness constant k. The expression of k is as follows

$$k = \frac{\pi^4 E_c I_c}{L^4} = \frac{\pi^4}{L^4} * (\textit{flexural rigidity}) \qquad (5)$$

Considering the effects of different centroid positions for the concrete deck slab and the steel I-section, Eq. (5) will be as follows

$$k = \frac{\pi^4 E_c I_c}{L^4} + \frac{\pi^4 E_s}{L^4} A_s e^2 + \frac{\pi^4 E_c}{L^4} A_c (h-e)^2 \qquad (6)$$

where e = distance between the centroid of the steel I-section and the centroid of the whole composite section, and h = distance between the centroid of the concrete deck slab and the centroid of the whole composite section.

The application of the proposed procedure is presented in the next section by means of a numerical example.

5. NUMERICAL EXAMPLE

Consider a simply supported plate girder of the cross-section shown in Fig. 3. Two values of the concrete strength have been considered for comparing the results with the present code [1].

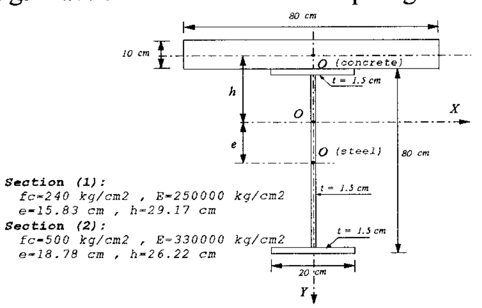

Fig. 3. The considered cross-section

Table 1. Results of the Proposed Procedure

L (m)	P_{cr} $(n = 7)$	P_{cr} (1)	P_{cr} (2)
1.0	600352362	570634272	619470319
2.0	150088091	142658568	154867580
3.0	66705818	63403808	68830035
4.0	37522023	35664642	38716895
5.0	24014094	22825371	24778813
6.0	16676455	1580952	17207509
7.0	12252089	11645597	12642251
8.0	9380505.7	8916160.5	9679223.7
9.0	7411757.6	7044867.6	7647781.7
10.0	6003523.6	5706342.7	6194703.2

The critical loads for in-plane instability of the simply supported plate girder with different lengths have been calculated based on Eq. (4) with substituting the values of the elastic stiffness constant of the spring from Eq. (6). From Table 1 it can be shown that the results of the present code are unsafe for section (1) and conservative for section (2).

6. SUMMARY AND CONCLUSIONS

A proposal for treating plate girder as an I-section member with uniformly distributed elastic springs of stiffness constants representing the rigidity of the deck slab is presented. The basic differential equations for the stability of the plate girder under in-plane loading condition is also presented. A formula for estimating the elastic stiffness constant of the springs for in-plane instability check is derived. The concept adopted in the present code of fixing value of n is compared with the proposed procedure and found to be either unsafe or conservative in most cases.

7. REFERENCES

[1] Japan Roadway Association, Roadway bridges specifications and commentary, Maruzen, Tokyo, 1994.
[2] Vlasov, V.Z. Thin-walled elastic beams, 2nd edition, Israel Prog. for Sc. Trans., Jerusalem, 1961.