CS - 12 Constitutive Modeling and FEM Analysis of Engineered Cementitious Composites

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1. INTRODUCTION: It has been shown by Li [2], that cementitious composites with short fibers can exhibit ductile behavior under tensile load, when the volume fraction of fibers exceeds a certain critical level. The critical fiber volume fraction depends on the matrix properties, fiber properties and fiber/matrix interaction properties. The mechanism of the pseudo strain-hardening, as this behavior is called, can be clarified when observing the response of the engineered cementitious composite (ECC) to uniaxial tension. After formation of the first crack at a loading equal to the first crack stress σ_{fc}, bridging fibers continue to transmit increasing stress across the crack. This leads to formation of additional subparallel cracks - a process called multiple cracking. The hardening response continues until the maximum bridging stress σ_{mb} is reached on some of the crack planes. The fracture then becomes localized into this crack and the overall response is softening. In our research, we seek to model the behavior of ECC under general stress state using data from simple material tests. We use plasticity modeling for the pseudo strain-hardening when the material undergoes the multiple cracking, and discrete crack model for the localized fracture. An independent plasticity strain-hardening model is used to account for the material nonlinearity in compression.

2. PROPOSED MODEL:

Model for the material in the multiple cracking state: We assume that at any point of the material, a crack develops in a direction normal to that of the maximum principal stress when its magnitude reaches the value of the first cracking stress σ_{fc}. The normal crack opening increases with increasing transmitted normal traction. If the direction of the maximum principle stress changes, a new crack develops in the corresponding normal direction. When unloaded, the existing crack keeps its opening constant. We assume that the stress level at which the cracking is initiated (σ_{fc}) as well as the post-first-cracking hardening are both independent of the lateral stress. When the material undergoes the multiple cracking, the crack density is very high while the crack openings are very small. Such a material state can be well represented by the smeared model, where the material is assumed to be continuous and the opening displacements of the cracks are represented by an additional strain. The incremental theory of plasticity provides a suitable tool that can accommodate all the assumptions stated above. The additional strain is then called plastic strain. Assuming 2-D stress conditions, we define the yield function as:

$$F \equiv 1/2(\sigma_x^* + \sigma_y^*) + 1/2\sqrt{(\sigma_x^* - \sigma_y^*)^2 + (\sigma_{xy}^* + \sigma_{yx}^*)^2} - \sigma_{fc} = 0 \qquad \textbf{(1)}$$

where σ_i^* equals to $\sigma_i - h\epsilon_i^p$ for $i = x, y, xy, yx$; σ_i are components of the stress vector; ϵ_i^p are components of the plastic strain vector; and h is a hardening coefficient. Note that the first two terms in Eq. **(1)** correspond to the magnitude of the maximum principal stress in the x-y plane, which means that the yielding will start when the magnitude reaches the value of σ_{fc} and that the plastic strain increment will occur in a direction parallel to that of the maximum principal stress. Note also that we use the linear kinematic hardening rule, which accounts for the assumption that the cracks at a certain point develop independently for different directions of the principal stresses. Both parameters σ_{fc} and h can be easily obtained from the direct tension test.

Model for localized tensile fracture: We assume, that the localized crack is initiated at the location and direction with the most developed damage. Therefore the localization starts at the point where the magnitude of the maximum tensile principal plastic strain reaches a critical value ϵ_{mb}^p, and the direction of the localized crack is normal to that of the maximum principal plastic strain. The localized crack is modeled as a discontinuity in the displacement field. The displacement gap in the direction normal to the crack is related to the traction transmitted in the same direction by means of a tension softening relationship. Both the parameter ϵ_{mb}^p and the tension softening relationship can be obtained from the direct tension test.

Model for nonlinearity in compression: To represent the material nonlinear behavior in compression, we use a strain-hardening plasticity model with Drucker-Prager yield function and isotropic hardening rule. The hardening rule is independent of the hardening due to multiple cracking, which means that there are two plastic

strain vectors - one for the nonlinearity in compression and another for the multiple cracking.
3. NUMERICAL RESULTS: The present material model has been implemented into the FEM code. We use isoparametric 4-node elements for the continuous domain and domain undergoing multiple cracking, while the localized crack is modeled by cracked element (see Nanakorn [3]). As the constitutive relations are in incremental form, we use the Euler method to obtain solution for total quantities. In order to verify our model we tried to reproduce experimental results published by Li and Hashida [1]. The paper provides results for two types of test specimens: uniaxial tensile specimen and double cantilever beam (DCB) specimen. All specimens were made from the same material: polyethylene fiber reinforced mortar with fiber volume fraction 2%. We used the data from the uniaxial test to set the material constants and with these constants we reproduced the test results for DCB. The computed load-displacement curve is shown in Fig. 1. Before the bend over point of the load-displacement curve, the cracked zone is very small and concentrated near the notch tip. Fig. 2 shows the state of the specimen at loading corresponding to point A on the load-displacement curve. We can see that, once the hardening portion of the load-displacement curve is reached, the damage spreads around the notch tip in the form of subparallel microcracks. We observed, that in most of the elements undergoing multiple cracking, a minimum tensile stress also reaches the level of the first cracking stress σ_{fc}, but very little plastic strain develops in its direction. At this stage, the localized crack already exists and propagates straight ahead of the original notch. At the side of the specimen opposite to the notch, compressive yielding takes place in a narrow area near the surface. From point A the overall response is almost linear hardening. As the load increases, the cracked region grows and the main localized crack propagates along the symmetry line. The specimen finally fails due to compressive failure at the surface opposite to the notch .
4. CONCLUSION: Referring to Li and Hashida [1] we can see that, as far as the direction of the microcracks and the shape and extent of the microcracking zone are concerned, our model can adequately reproduce the behavior of the material as it was observed in the experiment. Fig. 1 suggests that the trend of the load-displacement curve is also well represented by our model. Our prediction is, however, stiffer than the experimental result, which may be attributed to neglecting the influence of lateral stress on the hardening response of microcracks.

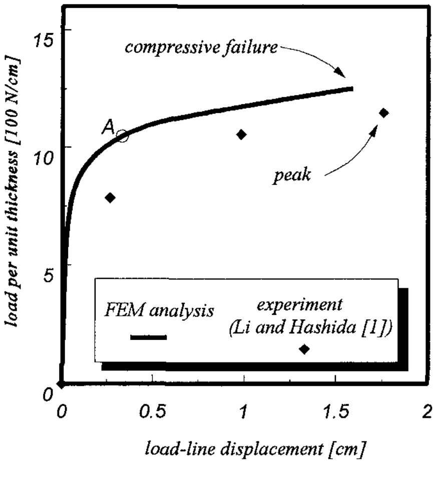

Fig. 1 Analytical and experimental load-displacement curve

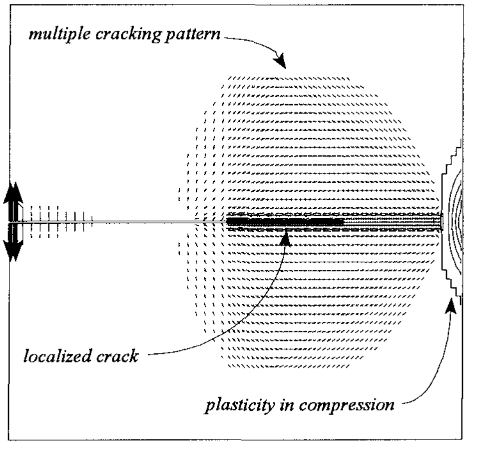

Fig. 2 State of specimen at point A of the load-displacement curve

5. REFERENCES:

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[2] Li, V.C., From micromechanics to structural engineering - the design of cementitious composites for civil engineering applications, *Structural Eng./Earthquake Eng. Vol.10, No.2, July 1993 (Proc. of JSCE No.471/I-24)*

[3] Nanakorn, P., Fracture mechanics based design method of SFRC tunnel lining, *Doctoral Dissertation, University of Tokyo 1993*