

## V - 416

## A New Method for Optimal Shape Design of Structures

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## Introduction

Up to date, the researches working on optimal shape design of structures are still difficult to be generalized for all kinds of material. Most of the design methods are under the assumption of equal compressive and tensile load response. However, for concrete-like or FRP-like material, their responses for compressive and tensile load are observed to be extremely unequal. Such effect of load response was seldom considered in the past studies. Hence, in this study, the authors try to propose a simple and generalized method for all preliminary optimal design for various kind of isotropic material within elastic limit even when their compressive and tensile strength are even or not.

## Description of Analysis

*a. optimization technique*

For optimizing the shape of structure in the case of a static elastic body, a method using a constitutive equation of growth in ref.[1] called "Incremental Growth Analysis (IGA)" is utilized. The characteristics of IGA are described fully in ref.[1].

*b. equivalent stress*

The calculation of equivalent stress (which is abbreviated as EQS) is different from the one used in ref. [1]. In ref. [1], EQS is expressed in term of octahedral shear stress. In this paper, the concept of EQS in two dimensional stress state is derived from the proposal of Dr. Okamura and Maekawa (ref.[2]), which defines EQS as a conceptual length of stress vector or distance between the origin and the stress point in the stress space. The main reason for adopting the latter is the easiness in programming and strength failure judgement.

Though EQS in ref.[2] is considered only on the stress state of compression-compression field, their proposal is used for all the biaxial-stress state within elastic limit.

*c. elastic limit failure*

The yield condition defines the elastic limit of a material under combined states of stress. In decades, lots of models for describing yielding condition are suggested. For expressing EQS, the distance from origin to elastic limit envelope will be normalized into unity by dividing dominant strength parameter ( $F_d$ ). The states of stress within elastic limit with same strain density energy can be formed into contours similar to elastic limit envelope, as illustrated in fig(1). And the value of EQS for any inner envelope would be between zero and unity. If the calculated EQS exceeds unity, it means elastic failure occurs within the structure.

*d. load response*

For most types of ductile material, their response for compressive and tensile load can be regarded as equal; however, for concrete-like brittle material, their response for compressive and tensile load has significant difference. It ranges from 7:1 to 20:1 or more. This is the reason why few researches has been done on optimal shape design with such kind of material. To integrate this load response into analysis, an idea for considering uneven load response of material in expressing EQS is suggested. The concept of the idea is schematized in fig(2). This idea is to assume the mapping relationship from uneven response envelope to even response envelope linear.

## Examples of Analysis

To illustrate the availability of this method, two analytical examples are carried out to examine its effectiveness.

*a. simple supported beam*

As shown in fig(3)(a-b), under the same initial configuration of loading, due to different constitutive equations of material, the optimized shapes of structure show apparent variation. Also, from table (1) and fig(3)-a, ( $S$ : width of fix edge of structure,  $D$ : depth of midspan,  $A$ : area of structure,  $Maxstr$ : maximum EQS in the structure), the effects of optimization on these constituent material are compared. Material 1, whose elastic limit envelope is assumed by Von Mises criteria, compared to Material 2 ( $F_c:F_t$

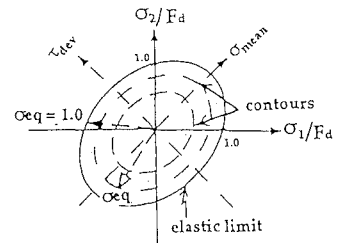


fig.(1) schematic diagram of EQS and elastic limit envelope

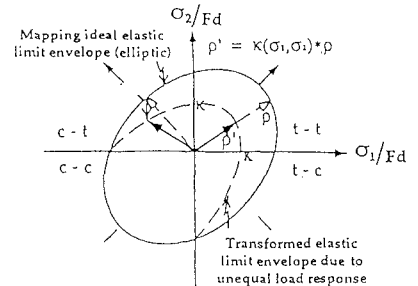


fig.(2) schematic diagram of integrating unequal load response into EQS

=2:1), and 3( $F_c:F_t=10:1$ ), whose elastic limit envelope is assumed by modified Von Mises criteria used in this paper, has the smallest midspan depth, smallest width of fix edge of structure due to its highest tensile load capacity.

#### b. cantilever beam

In fig(4)(a), a comparison is made between Material 3 ( $F_c:F_t=10:1$ ) and 4 ( $F_c:F_t=1:10$ ), as shown in the figure ( $R_t$ : the vertical length of the part of cantilever beam being-connected to the wall,  $E_d$ : the vertical length of the free end of cantilever beam,  $u$ : upper part and  $d$ : lower part as divided by neutral line  $P_1$ : the area of upper part and  $P_2$ : the area of lower part as divided by neutral line). The structures made of M3 and M4 are optimized to the opposite configuration ;that is,  $P_1$  of M3 is smaller than the one of M4 and  $P_2$  of M3 is greater than the one of M4. It indicates that the difference results from the variation of dominant material strength parameter in both cases.

Material Type	Material 1	Material 2	Material 3
Material Strength Parameters & Failure Criteria			
Failure Criteria	Von Mises criteria	modified Von Mises criteria	modified Von Mises criteria
$F_c : F_t$	1 : 1	2 : 1	10 : 1
Initial Configuration of Loading : Initial Dimension of Specimen =>			
loading type : simple supported beam	cross section 15 * 15 (cm) length = 120 cm span = 110 cm		
Load (P) = 200 kgf	dominant strength parameter ( $F_c$ ) = 140 kgf/cm <sup>2</sup>		
$A/A_0$	1.091833	1.120017	1.140661
$D/D_0$	1.11454	1.11490	1.11722
$S/S_0$	1.91741	1.94934	1.95689
Maxstr/Maxstr <sub>0</sub>	0.638150	0.627745	0.621964

in which  $A_0 = 0.15m^2$ ,  $D_0 = 0.15m$ ,  $S_0 = 0.10m$ , Maxstr<sub>0</sub> = 0.270178

Table (1) Comparison of Optimization Efficiency on M1, M2 & M3 in the case of simple supported beam

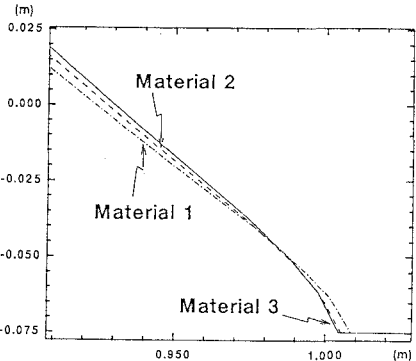


Fig.(3)-b Magnification View around fix edge of structure

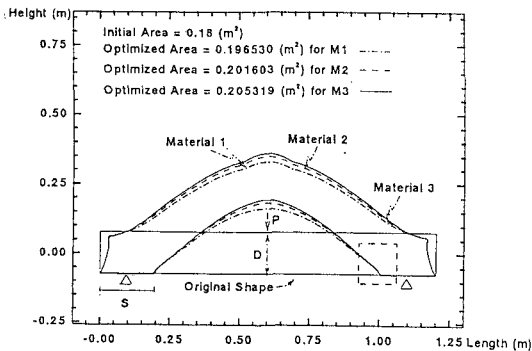


Fig.(3)-a Result of Optimization in the case of simple support beam

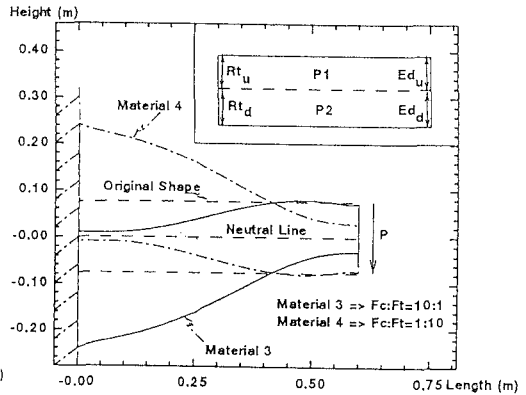


Fig.(4)-a Result of Optimization on cantilever beam

## Conclusions

The following conclusions can be extracted from this study:

- (1) The distribution of stress within structure can be effectively reduced to be more uniform.
- (2) Compared to previous optimal shape design techniques, the method proposed in this paper may be more comprehensive because of its availability on material with significant difference on compressive and tensile strength. This is confirmed by the previous examples.
- (3) Because the idea of including unequal load response into the calculation of EQS is simplified in this paper, a more sophisticated formula for describing the transformation(mapping) relationship must be investigated to describe the failure envelope precisely.

## References

- [1] Azegami, H., "A Proposal of a Shape-Optiization Method Using a Constitutive Equation of Growth (In the Case of a Static Elastic Body)", JSME Inter. J., Ser. I, Vol.33, No.1, 1990, pp. 64-71.
- [2] Maekawa, K., and Okamura, H., "The Deformational Behaviour and Constitutive Equation of Concrete Using the Elasto-Plastic and Fracture Model", J. of the Fac. of Engrg, Univ. of Tokyo (B) Vol. XXXVII, No.2, 1983, pp. 184-251.