IV - 457

On the derivation of realized level of utility as a function of population - General equilibrium modelling in a system of two cities -

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Introduction

Unbalanced distribution of population among cities has been regarded as the most serious issues in nationwide spatial planning and policies. Most of theoretical studies analyzing population distributed in a system of cities, motivated with the above background, have described equilibrium states where none has an incentive to relocate. To define such states, realized level of utility should be derived as a function of population (in the rest of the paper, we call it the Function). Furthermore, some properties of the Function result in interesting states such as multiple-equilbira, and "low utility trap" (Sakashita(1989)). Panel (a) in Figure 1 shows an example that the Function with a peak results in multiple equilibra, compared with Panel (b). However, the Function has been assumed a priori with poor microfundation for it.

UEDA(1993) showed many examples of the Function, curves of which were drawn by numerical computation, and Morisugi et'al (1993) listed up factors which would be dominant to the Function. However, these studies are still at preliminary stage of research, unsuccessful in getting the Function analytically. This paper is a note on the derivation of realized level of utility as a function of population, that is, the Function based on a full setting of Walrasian general equilibrium in a system of two cities.

Model

Sketch: major assumptions are, i) an economy consists of two city in a nation (denoted by i and j) and the rest of the world. i) there are a fixed number of households with an identical preference, a representative firm in each city, and an absentee landowner. iii) free mibility of households within the nation is assumed. iv) the transport cost for trading of goods is iceberg type. v) a kind of externality, knowledge spill over, is considered in the production of goods. vi) each city specialises in production of one goods (denoted by i and j).

Utility maximization of a household : household's behavior in city i is formulated as,

$$V_i = \max_{z,q} \alpha_i \ln z_i + \alpha_j \ln z_j + \beta \ln q_i \qquad (1.a)$$

$$s.t. \quad p_i z_i + \left(\frac{p_j}{\tau_{ji}}\right) z_j + r_i q_i = w_i \qquad (1.b)$$

We note that, V; indirect utility, z; consumption of goods, q ; consumption of land, p; f.o.b price of goods, r; land rent, w; wage income, $\alpha_i + \alpha_j + \beta = 1, \alpha_i > 0, \alpha_j > 0, \beta > 0$; preference parameters, τ_{ji}; remaining rate of goods after transport from j to i . Here, the higher remaining rate is, the lower iceberg transport cost is, and τ_{ii}. is normalized to be 1. From the F.O.C. of (1) and some manipulations, we have demand functions and an indirect utility function as,

$$z_i = \alpha_i \left(\frac{w_i}{p_i}\right) \text{(2.a)}, \quad z_j = \alpha_j \left(\frac{w_i \tau_{ji}}{p_j}\right) \text{(2.b)}, \quad q_i = \beta \left(\frac{w_i}{r_i}\right) \text{(2.c)}$$

$$V_i = \ln w_i - \alpha_i \ln p_i - \alpha_j (\ln p_j - \ln \tau_{ji}) - \beta \ln r_i + const. \quad (3)$$

Profit maximization of a representative firm: firm's behavior is formulated as,

$$\pi_i = \max_{Z,N,L} p_i Z_i - w_i N_i - R_i L_i \qquad (4.a)$$

$$s.t. \quad Z_i = A_i N_i^{\gamma_i} L_i^{\delta_i} \qquad (4.b)$$

Notations are, here, π; indirect profit, Z; production of goods, N;labor input, L;land input, R;land rent for firm, A;level of production technology, $\gamma + \delta = 1, \gamma > 0, \delta > 0$, ; technology parameters. From the F.O.C. of (4), we have,

$$w_i = \gamma_i \, p_i A_i N_i^{\gamma_i - 1} L_i^{\delta_i} \qquad (5.a)$$

$$R_i = \delta_i \, p_i A_i N_i^{\gamma_i} L_i^{\delta_i - 1} \qquad (5.b)$$

$$and\ automatically, \pi_i = 0 \quad (5.c)$$

Walrasian multimarket equilibrium: we assume that land supply in each city is fixed, and that any household supplies labor normalized to be 1, then market clearing conditions are,

$$Z_i = N_i \alpha_i \left(\frac{w_i}{p_i}\right) + N_j \alpha_i \left(\frac{w_j \tau_{ij}}{p_i}\right) + C_I \alpha_i \left(\frac{\tau_{Ii}}{p_i}\right) \quad for\ i = 1,2 \qquad (6.a)$$

$$N_i \alpha_i \left(\frac{w_i}{r_i}\right) = l_i \quad for\ i = 1,2 \qquad (6.b)$$

and (5) for i=1,2.

Here, C ; potential demand in the rest of the world, and subscript I labels the rest of the world. The first equation in the above is balance of aggregate demand and supply in goods market, the second is in land market as well. The condition for goods market, (6.a) includes intercityal and international trade, while the land is exclusively traded only within a city, as in (6.b). Condition of full employment of labor and in each city have to be consistent with marginal productivity equation in (5.a). Therefore, labor input N_i is regarded as population in the above conditions.

Externality : As is well known, agglomeration may bring many kinds of merits to any firms locating in a city, while accompanied with negative effects like high land rent or degrading of environmental quality. Such merits are, i) agglomeration raises up level of knowledge and therefore technology, though spill over process with face-to-face contact. ii) agglomeration leads to the variety of skilled labors and intermediate input goods, and therefore flexible structure of production, which are not explicit in the above formulation. Recent theories of endogenous economic growth often assume that the total of accumulated capital is a proxy of such positive effect on level of production technology. This is because of a line of thought that knowledge is embodied into the capital. However, since in this paper, capital has not appeared in the model in this paper, the knowledge potentially raising up technology is assumed to be embodied in labors. Then, we have to model that population agglomerated in a city is a proxy of such an externality. Here we specify,

$$A_i = N_i^{\varepsilon_i} \qquad (7)$$

Derivation of realized level of utility

as a function of population

Realized level of utility has been already formulated as an indirect utility function in (3). To express it as a function of population, first, let us fix population variables, N in (5) and (6) and solve them with respect to price variables, p, w and we get an unique solution of them as function of population variables, N. Then, inserting them and also (7) into (3), we have, with some arrangements,

$$V_i = \{\alpha_i(\gamma_i+\varepsilon_i)-1\}\ln N_i + \alpha_j(\gamma_j+\varepsilon_j)\beta \ln N_j + \alpha_j\left[\ln\left\{\frac{\alpha_j\gamma_j C_i\tau_i}{(1-\alpha_j\gamma_j)}\tau_i + C_i\tau_i\right\} - \ln\left\{\frac{\alpha_i\gamma_i C_i\tau_i}{(1-\alpha_i\gamma_i)}\tau_j + C_j\tau_j\right\}\right] + \alpha_i \ln L_i + \alpha_j \ln L_j + \alpha_j \ln\tau_j - \beta_i \ln l_i + const. \quad (8)$$

Properties of the Function

Maximum: Among properties of the Function, it is the most important point whether or not the Function has a peak in a specific domain, as already shown. To examine it, here we add the following assumption,

$$N_i + N_j = N_T \quad (9)$$

which is the constraint of total population in a nation. With the above assumption, we have,

$$\frac{\partial V_i}{\partial N_i} = \frac{\alpha_i(\gamma_i+\varepsilon_i)-1}{N_i} - \frac{\alpha_j(\gamma_j+\varepsilon_j)}{N_T - N_i} \quad (10)$$

From this, the condition necessary and sufficient for the existence of maximum is,

$$\alpha_i(\gamma_i+\varepsilon_i)-1>0 \quad (11)$$

and, N giving it is,

$$N_i = \frac{\alpha_i(\gamma_i+\varepsilon_i)-1}{\alpha_i(\gamma_i+\varepsilon_i)-1+\alpha_j(\gamma_j+\varepsilon_j)}\cdot N_T \quad (12)$$

When the condition holds, we know that,

$$V_i \to -\infty, \quad as\ N_i \to 0 \quad (13.a)$$

$$V_i \to -\infty, \quad as\ N_i \to N_T \quad (13.b)$$

The Function in the above case is depicted in Panel (a) of Figure 2. If the condition does not hold, then the Function is monotonously decreasing, and it has properties as,

$$V_i \to \infty, \quad as\ N_i \to 0 \quad (14.a)$$

$$V_i \to -\infty, \quad as\ N_i \to N_T \quad (14.b)$$

This case is shown in Panel (b) of Figure 2.

Economic Interpretation : What is an economic interpretation that the condition (11) gives ? In form of the Function, it means that coefficient of the first term in the RHS of (8) is positive. The household's utility increases as the population in its own city, in other words, the agglomeration becomes larger. However, on the other hand, the decrease of the population in the other city reduces the utility level though the second term in the RHS of (8). The condition (11) depends on parameters, α_i, γ_i, and ε_i. The greater these parameters are, the more possibly the condition holds. α_i is the preference parameter denoting the weight of expenditure for the goods, the production of which the city specializes in. Thus, the greater α_i means large potential demand in the economy. γ_i determines the productivity of labor input. It is natural that the greater productivity realizes higher level of utility. Since ε_i is the parameter denoting intensity of the externality in (7), its higher value raises up the utility though level of production technology as already mentioned. By some manipulations, we can know that the condition (11) implies that,

$$\gamma_i + \varepsilon_i > 1 \quad (15)$$

Inserting (7) into (4.b), we have,

$$Z_i = N_i^{\gamma_i+\varepsilon_i} L_i^{\delta_i} \quad (16)$$

The condition (15) means, in form of (16), that the production technology shows increasing return to scale with respect to labor input at aggregate level. However, at micro state of production, each firm takes the level of A as given, and then, has zero profit because of linear homogeneous technology represented by $\gamma+\delta=1$. Then, the condition (15) is consistent with the assumption of perfect competition in markets.

Concluding remarks

This paper shows the derivation of realized level of utility as a function of population, based on a full setting of Walrasian multimarket equilibrium. Although this derivation itself, of course, directly gives no political or planning implications, because of micorfundation, it gives clear economic interpretations because of micorfundatio,. The Function shown here will be installed into the impact analysis of cityal policies on a system of cities, in the stage next to this study. It is needless to say that factors explicitly considered in the Function like available land, transport cost, productivity, and so on can represent major cityal policies. In the other paper, we are intending to show outcomes of the next stage of research in this line.

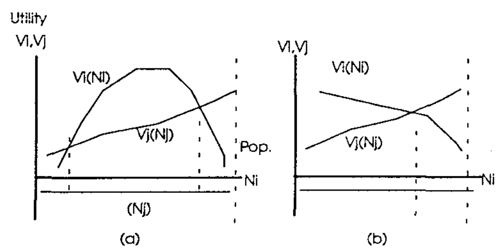

Figure 1 Example of multiple-equilibra

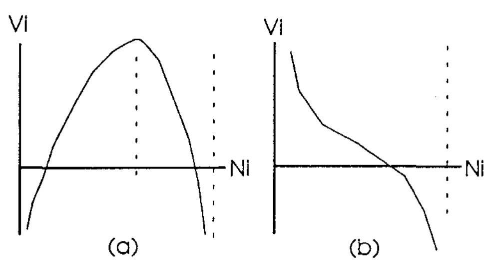

Figure 2 Curves of the Function

Referrence

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