IV-451 Interregional Freight Flow Analysis Based on Rectangular Input-Output Tables Using the Lagrangian Multiplier Approach

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Abstract : This paper proposes the use of the multi-regional rectangular input-output (MRRIO) table for interregional freight flow analysis. There are two main steps. The first is to prepare the initial MRRIO table from the multi-regional input-output table (Leontief form) and the national rectangular output (V) table. The second is to correct the initial table to meet row and column constraints. Finally, the frequency and percentage of amount of estimated values for each absolute percentage deviation levels are presented.

1. Introduction

The rectangular model is superior to the Leontief model in that it is more exhaustive and because it handles the secondary-product problem more effectively [1]. Moreover, based on various single-region applications, the rectangular system can be constructed to the multi-regional model.

The procedure was applied to the 1985 multi-regional input-output table (Leontief form) and 1985 national rectangular output (V) table of Japan to construct the MRRIO model. The nation was divided into three (n) regions, i.e., north-eastern (r), center (s) and others (o). Commodity and industry were classified into ten (p) categories.

2. Methodology

TABLE 1: The MRRIO Model

	com	ind				com	ind				
com		U^{r}_{ij}	f^{rr}_{i}	e^{r}_{i}	h^{r}_{i}		U^{r}_{ij}	f^{rs}_{i}		z^{r}_{i}	q^{r}_{i}
ind	V^{r}_{ij}					V^{r}_{ij}					g^{r}_{i}
	h^{r}_{j}					z^{r}_{j}					
com		U^{s}_{ij}	f^{sr}_{i}		z^{s}_{i}		U^{s}_{ij}	f^{ss}_{i}	e^{s}_{i}	h^{s}_{i}	q^{s}_{i}
ind	V^{s}_{ij}					V^{s}_{ij}					g^{s}_{i}
	z^{s}_{j}					h^{s}_{j}					
		y^{r}_{j}					y^{s}_{j}				
		g^{r}_{j}					g^{s}_{j}				

U : input matrix, V : output matrix
q : domestic input vector
h : inter-regional input vector
z : intra-regional input vector
g : industry output vector
f : final demand
e : net national export
y : value added
r, s : region
i, j : commodity or industry sector
com : commodity, ind : industry

2.1 The Initial MRRIO Table

From the system of national accounts;

(1) $q = (I - BC^{-1})^{-1} (f + e)$

I : unit matrix

B: input coefficient matrix with dimension of commodity x industry

C: output coefficient matrix with dimension of commodity x industry

From interregional input-output analysis (Leontief form);

(2) $q = (I - TA)^{-1} (f + e)$

T : trade coefficient

TA : coefficient matrix with dimension of commodity x commodity

From Equations (1) and (2);

(3) $TA = BC^{-1}$

It is assumed that each inter-regional C coefficient structure and each intra-regional C coefficient structure are the same as the national C* coefficient structure. The initial MRRIO table was constructed as shown in Figure 1.

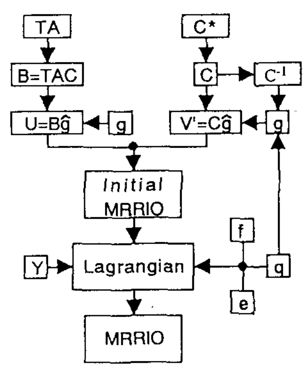

FIGURE 1: Overview of Methodology

2.2 The Estimated MRRIO Table

The Lagrangian multiplier approach was employed to minimize the weighted squared sums of deviations between the entries in the initial matrix and estimated matrix. The general form [2] can be expressed as follows:

TABLE 2: The Frequency Distribution of Absolute Percentage Deviations

Estimated	0-0.9	1-4.9	5-9.9	10-19.9	20-49.9	50-99.9	100-199	>=200
	Absolute Percentage Deviations							
U	13	47	49	143	310	230	108	0
V	336	13	19	27	34	29	84	358

$$\mathrm{Min} \sum_{ij} \frac{(\tilde{X}_{ij} - X_{ij})^2}{X_{ij}}$$

$\tilde{X}_{ij}$: entry in estimated U or V matrix

X_{ij} : entry in initial U or V matrix

The subjective functions of U are as follows:

$$\sum_m \sum_i \tilde{U}^{mr}_{ik} + y^r_k = g^r_j \quad (j=1,...,n(p-1);\ \text{column constraints})$$

$$\sum_j \tilde{U}^{rs}_{kj} + f^{rs}_k = z^r_i \quad (i=1,...,pn(n-1);\ \text{row constraints})$$

$$\sum_j \tilde{U}^{rr}_{kj} + f^{rr}_k + e^r_k = h^r_i \quad (i=1,...,pn;\ \text{row constraints})$$

The augmented objective function of U is

$$Lu = \sum_{ij} (\tilde{U}_{ij} - U_{ij})^2/U_{ij} - \sum_{j=1}^{n(p-1)} \beta_j\left(\sum_m \sum_i \tilde{U}^{mr}_{ik} + y^r_k - g_j\right) - \sum_{i=1}^{pn(n-1)} \phi_i\left(\sum_j \tilde{U}^{rs}_{kj} + f^{rs}_k - z_i\right) - \sum_{i=1}^{pn} \alpha_i\left(\sum_j \tilde{U}^{rr}_{kj} + f^{rr}_k + e^r_k - h_i\right)$$

differentiating Lu;

$$\frac{\partial Lu}{\partial \tilde{U}_{ij}} = \frac{2\tilde{U}_{ij} - 2U_{ij}}{U_{ij}} - \beta_j - \phi_i - \alpha_i = 0 \tag{4}$$

The subjective functions of V are as follows:

$$\sum_m \sum_j \tilde{V}^{rm}_{kj} = g^r_i \quad (i=1,...,n(p-1);\ \text{row constraints})$$

$$\sum_i \tilde{V}^{rs}_{ik} = z^s_j \quad (j=1,...,pn(n-1);\ \text{column constraints})$$

$$\sum_i \tilde{V}^{rr}_{ik} = h^s_j \quad (j=1,...,pn;\ \text{column constraints})$$

The augmented objective function of V is

$$Lv = \sum_{ij} (\tilde{V}_{ij} - V_{ij})^2/V_{ij} - \sum_{i=1}^{n(p-1)} \mu_i\left(\sum_m \sum_j \tilde{V}^{rm}_{kj} - g_i\right) - \sum_{j=1}^{pn(n-1)} \theta_j\left(\sum_i \tilde{V}^{rs}_{ik} - z_j\right) - \sum_{j=1}^{pn} \gamma_j\left(\sum_i \tilde{V}^{rr}_{ik} - h_j\right)$$

differentiating Lv;

$$\frac{\partial Lv}{\partial \tilde{V}_{ij}} = \frac{2\tilde{V}_{ij} - 2V_{ij}}{V_{ij}} - \mu_i - \theta_j - \gamma_j = 0 \tag{5}$$

From Equations (4) and (5), 1800 equations are formulated. Together with the 234 constraints, 1800 entries of U and V can be estimated.

TABLE 3: The Percentage of Amount of Estimated Values for Each Absolute Percentage Deviation Levels

Estimated	0-19.9	20-49.9	>=50
	Absolute Percentage Deviations		
U	63.1	33.3	3.6
V	91.6	3.5	4.9

3. Results and Discussion

The frequency and the percentage of amount of estimated values for each absolute percentage deviation levels are shown in Tables 2 and 3, respectively. The absolute percentage deviation is given as;

$$d_{ij} = \frac{|\tilde{X}_{ij} - X_{ij}|}{X_{ij}}; \quad X_{ij} \neq 0$$

Most of the estimated U values have the absolute deviations between 20 and 100% while most of the estimated V values have the absolute deviations which are either less than 1% or more than 200%. It was also noted that the estimated values which are small in magnitude have large absolute deviations. The number of negative values are 38 and 190 for estimated U and V, respectively. However, 63.1% and 91.6% of the amount of estimated U and V, respectively, have absolute deviations of less than 20%.

It was found that this method can be applied to real world. However, revision of this method is required to reduce errors in small items and to reduce the number of negative values. This method was applied to the small matrix, it should be expanded to the larger necessary matrix.

4. References

[1]Louis, L.V.St, 1989."Empirical Tests of Some Semi-Survey Update Procedures Applied to Rectangular Input-Output Tables", Journal of Regional Science, 29, 273-385.

[2]Morrison, W.I. and R.G.Thumann, 1980."A Lagrangian Multiplier Approach to the Solution of a Special Constrained Matrix Problem," Journal of Regional Science, 20,279-292.