III- 430 Numerical schemes in indirect boundary element method

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1. Introduction

One of the main problems of tunnelling consists in working out measures ensuring a required stability of a rock mass in the vicinity of tunnels. To solve this problem it is essential to have accurate and operative estimations of tunnels stability both during construction and after completion. Now different numerical methods are widely used for this purpose, and the Boundary Element Method (BEM) is one of them. This method is not so universal as the Finite Element Method (FEM), but its utilization needs the discretization only of the boundary of a domain considered for analysis. This gives possibility to decrease significantly the cost of input data preparation, the required computer memory and the time of calculation in comparison with those required for the FEM and makes the BEM very convenient for practical engineering.

One of the crucial steps in efficient BEM programming lies in the numerical integration of concentrated forces on boundary elements. Since the solution for the concentrated force has a singularity at the point of application, then usually the BEM allows to receive rather exact values of stresses and displacements at points distant from the boundary, while near the surface because of numerical errors the BEM solution is not available for practical use. It is possible to diminish scale of the region where stresses and displacements are calculated with errors by decreasing the size of boundary elements or by increasing the number of integration points on elements. But this could result in the matrix size not manageable for computer memory or could increase the computation time such that the cost will be extremely high especially in 3-dimensional case (crossing tunnels, system of tunnels, etc.). So it is expedient by taking into account the features of each class of problems to create the calculation schemes that allow us to produce exact enough results for minimum computation time.

2. Numerical schemes

The tunnel section with a length 10 *m* and circular cross-section (the radius was equal to 1 *m*) was considered. The rock mass was assumed as elastic isotropic medium. In the virgin rock mass the stress state was hydrostatic. Stresses and displacements were calculated by the BEM and analytically. For values of radius and the length considered in the middle cross-section of the tunnel, stresses and displacements must agree, within several percentage of errors, with those given from analytical solution of plane problem for the tunnel of infinite length[1,2].

In accordance with the indirect BEM[3,4] it was assumed, that the space inside of the tunnel was filled up of material with the same elastic properties as the rock mass. That is the infinite homogeneous medium without any cavity was considered. The tunnel surface approximation was made by identical flat rectangular boundary elements. So each element, representing a part of the surface as a geometrical object, was situated in the infinite homogeneous medium with elastic properties of the rock mass. This trick allowed us to use known fundamental solutions for loads acting in an infinite medium. Assuming, that evenly distributed load acted on every boundary element, the problem was reduced to finding out such loads distribution that satisfied the boundary conditions. Thus appropriate loading of boundary elements allowed us to model the tunnel presence, when stresses and displacements in the rock mass with the tunnel were the same as in infinite elastic medium without tunnel, but with loaded boundary elements.

Two methods were used to diminish the time of calculation by the BEM:

1. Forming the main system of equations with predominant diagonal coefficients and using iterative methods for solving.

2. Flexible scheme for numerical integration of concentration forces. Each boundary element was divided into several subelements. The amount of subelements on the boundary element depended on the distance between the boundary element and point under consideration. That is, for example, if the point under consideration was situated far from the boundary element and the element is small then the distributed load was substituted by a single force. And if the distance was shorter comparing with the element size then the element was divided into subelements. This flexible scheme of calculations allowed us to receive exact enough numerical solution of the problem and to diminish the calculation time, because only part of the total boundary elements were divided into subelements.

3. Results

Use of the local coordinate systems connected with each boundary element has allowed us to form matrix of the main system of equations with predominant diagonal coefficients and utilized for the system solving the rapidly converging iterative method. The CPU time t_{solv} for solving the main system on a workstation (Sun Sparc 10) by different methods, via total amount of the boundary elements N, are shown below

N	24	120	240	480	2400
Gauss elimination method, t_{solv} , sec.	0.3	44.0	436.0	-	-
Iterative method, t_{solv} , sec.	0.07	1.5	6.2	37.0	870.0

Although calculations by the Gauss elimination method have not been done for $N = 480$ and $N = 2400$ the data obtained show clearly the advantage of using iterative method for solving the main system of equations.

To control the process of subdividing and to calculate the amount of subelements n_l along the long side of the boundary element ($n_l \geq 1$ was an integer number) two parameters C_{inf} and n_{max} were used

$$n_l = \begin{cases} 1, & \text{if} \quad C_{inf} \cdot (L_{max} / R) < 1 \\ C_{inf} \cdot (L_{max} / R) & \\ n_{max}, & \text{if} \quad C_{inf} \cdot (L_{max} / R) > n_{max} \end{cases}$$

Parameter n_{max} was used to define the maximum possible number of subelements along the long side of the boundary element. Parameter C_{inf} was used for correcting the subdividing process (if $C_{inf} \to \infty$ then all boundary elements were divided into the smallest subelements). Here R is the distance between the point under consideration and the boundary element ; L_{max} is length of long side of the boundary element. The amount of subelements along the short side of the boundary element was an integer number $n_s \geq 1$ which was defined as $(L_{min} / L_{max}) \cdot n_l$, where L_{min} is the length of the short side of the boundary element.

Utilization of the system of forces instead of the single force for modelling the influence of boundary elements has allowed us:

- To attain the convergency of the iteration procedure for coarse meshes and to use iterative method for rapid solution of the main system of equations ($t_{solv} = 436.0$ sec., Gauss elimination method; $t_{solv} = 6.2$ sec., iterative method for $N = 240$).

- To improve the accuracy of calculating stresses and displacements on the tunnel surface and at internal points (the maximum level of numerical errors for circumferential stresses, radial stresses and radial displacements was $\varepsilon = 125.3\%$ for $N = 2400$ and the single force; $\varepsilon = 7.9\%$ for $N = 24$, the system of forces, $C_{inf} = 5$ and $n_{max} = 200$), that enabled us to use the coarser mesh and as a result to decrease the size of the computer memory required and the time of forming the main system of equations (in the main system of equations the number of coefficients was equal to 5.184.000, the time required to form the main system of equations was $t_{form} = 995.1$ sec. for $N = 2400$ and a single force; in the main system the number of coefficients was equal to 5.184, $t_{form} = 6.1$ sec. for $N = 24$, the system of forces, $C_{inf} = 5$ and $n_{max} = 200$).

- To use the flexible scheme of calculation and as a result to decrease the duration of computation on 5 to 10 times keeping the accuracy required comparing with the schemes of numerical integration with a constant amount of subelements on each boundary element .

- To guarantee the accuracy of calculations within 10% with minimum duration of calculations if $C_{inf} = 5$ and n_{max} was defined by the formula

$$n_{max} = 2 \cdot L_{max} / R_{min} \quad ,$$

where R_{min} was the minimum distance from the point under consideration to the tunnel surface.

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