III-282 Studies on effect of various micro-structures on elastic moduli by homogenization method

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1. Introduction

The effective elastic properties of geomaterials are the subject of long standing interest in the mechanics community. Theoretical studies on effective elastic properties may be divided into three general classes: variational methods[1], self-consistent method[2] (S.Nemat-Nasser, M.Hori) and microscopic field method[3]. It is powerful for periodic micro-structures to employ homogenization method, which consider the problem in both micro-scale and macro-scale, to find out effective elastic properties. This method is based on the understanding of microscopic field and revise the macroscopic average field from microscopic heterogeneity of materials. In this paper, the expression of macroscopic equivalent matrix is composed of averaging gradient and micro-perturbation induced by micro-heterogeneity. The micro-perturbation is described by a local problem. Finally the theory is applied to the micromechanics of a concrete with various micro-structures and comparison to experimental data is made.

2. Fundamentals of homogenization method

2.1 Fundamental equations

The equivalent moduli of a unit cell is derived out based on homogenization theory[3]:

$$E^h_{ijkl} = \bar{E}_{ijkl} + \tilde{E}_{ijkl} \qquad (1)$$

Where $\bar{E}_{ijkl} = \frac{1}{|Y|}\int_Y E_{ijkl}\,dY$ is the volume average of every constituents controlled by volume fraction. $\tilde{E}_{ijkl} = \frac{1}{|Y|}\int_Y \left\{-\left(E_{ijpq}\frac{\partial W^{kl}_p}{\partial y_q} + E_{pqkl}\frac{\partial W^{ij}_p}{\partial y_q}\right) + E_{qnpm}\frac{\partial W^{ij}_q}{\partial y_n}\frac{\partial W^{kl}_p}{\partial y_m}\right\}dY$ is the fluctuation due to the interaction among constituents. It will be shown that it is the W^{kl}_p, called characteristic functions and determined by following local problem (Eq.(2)), that represent the interaction.

$$\frac{\partial}{\partial y_l}\left\{E_{klij}\left(\delta_{ip}\delta_{jq} - \frac{\partial W^{pq}_i}{\partial y_j}\right)\right\} = 0 \quad \text{With Y-periodicity condition on } W^{pq}_i. \qquad (2)$$

2.2 Some remarks on local problem

The weak form of Eq.(2) is written as:

$$\int_Y \frac{\partial}{\partial y_l}\left[E_{klij}\frac{\partial W^{pq}_i}{\partial y_j}\right]V_k(y)\,dY = \int_Y \frac{\partial E_{klpq}}{\partial y_l}V_k(y)\,dY \qquad (3)$$

where $V_k(y)$ is continuous in Y-periodicity and its derivatives exist. Let's see $Y = \bigcup_i Y_i \oplus \bigcup_{i,j}\Gamma_{ij}$ and E_{ijkl} are constants if $y \in Y_i$. Γ_{ij} is the interface of sub-domains Y_i & Y_j.

$\int_Y \frac{\partial E_{klpq}}{\partial y_l}V_k(y)\,dY = \sum_{i=1}^{n}\int_{Y_i}\frac{\partial E_{klpq}}{\partial y_l}V_k(y)\,dY + \sum_{i,j}\int_{\Gamma_{ij}}\frac{\partial E_{klpq}}{\partial y_l}V_k(y)\,d\Gamma$. The first term of right hand side equals to zero. The second term is:

$$\int_{\Gamma_{ij}}\frac{\partial E_{klpq}}{\partial y_l}V_k(y)\,d\Gamma = \left[E^{(i)}_{klpq} - E^{(j)}_{klpq}\right]\bar{V}_k(y)] \qquad (4).$$

Eq.(4) shows that the force sources of characteristic functions originate from the inhomogeniety of unit cell. $\bar{V}_k(y)$ is the average value of $V_k(y)$ on Γ_{ij}. It is easy to prove that this force is a self-equilibrated system. Otherwise, there is no unique solution.

3. Prediction of elastic moduli of concretes with different microstructures

3.1 Effect of location, distribution and shape of inclusion B

The effect of location on moduli is seen in Fig.2. Several cases have shown that effective moduli are determined by only volume fraction if the inclusion is translated in the unit cell. But the moduli are different from those obtained by volume fraction averaging technique. It is noticed that if inclusion B is divided and distributed in the unit cell the effective moduli are different. This implies that the distribution or interaction of inclusions has effect on macro-properties. Results rotating B in plane shows that shape of inclusions has vital effect on equivalent properties.

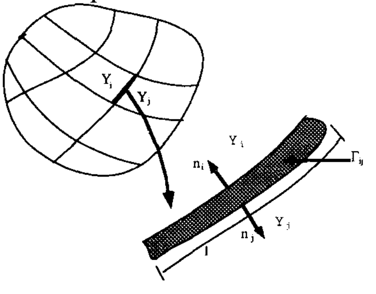

Fig. 1 Interface between Subdomains

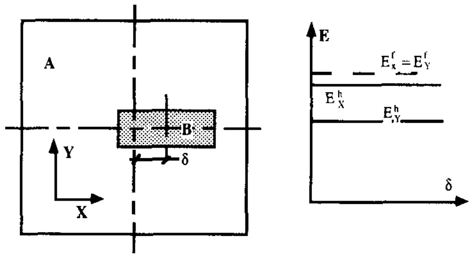

Fig.2 Effect of Inclusions on Mechanical Properties

3.2 Prediction of elastic moduli of concretes with different microstructures

CELL.FORT[4] was also used to predict the elastic moduli of a concrete tested by A.M. Farahat[5]. The periodic unit cell is assumed in Fig.3. The interface between inclusion and mortar are assumed to bond completely[4]. Theoretical values E^h_y are always larger than observed values E_y, but less than the micro-stiffness E_n (see Table 2), agreeing with the experiment. The details of above calculations are seen the full paper of this one.

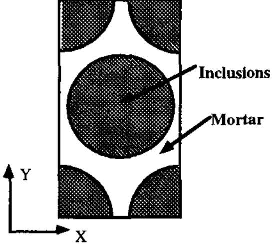

Fig.3 Unit Cell of A Concrete Sample

Table 1 Original Data

Inclusion	Steel Cylinder (S-C)	Granite Cylinder (G-C)	Mortar *
E (×10^4MPa)	18	5.99	2.07
μ	0.3	0.2	0.18

* Mortars are a little different in different cases. See [5].

Table 2 Homogenized E^h_y and Experimental E_y

Cases	Case 1 (S - C)	Case 2 (G - C)	Case 3 (G - C)
E^h_y(10^4MPa)	4.2	3	3.2
E_y(10^4MPa)	3.6	2.67	2.8
E_n(10^4MPa)	4.6	3.2	3.5

4. Conclusions

1) It has been shown that homogenized moduli, different from those by volume fraction average, are not sensitive to the location of a single inclusion. Volume fraction is the most important quantity. The distribution, shape and inclination of inclusions have vital effect on homogenized moduli.

2) The homogenized moduli for the concrete samples are always higher than those measured in experiment. This may be due to that CELL.FORT now considers the interfaces between inclusion and mortar as complete bonding. This is not true because of initial defects in concretes. Improvement is in progress to involve partial debonding and nonlinearity of each constituent.

References

[1] Hashin, Z. and Shtrikman, S.(1962), on some variational principles in anisotropic and nonhomogeneous elasticity, J. Mech. Phys. Solids, 10:335-342 [2] K.C. Nunan and J.B. Keller(1984), Effective elasticity tensor of a periodic composite, J. Mech. Phys. Solids, 32(4):259-280 [3]Jian-Guo, Wang and Y. Ichikawa(1994), One dimensional nonlinear model of sand densification by homogenization method, Proceeding of Chubu Regional Civil Engineering Meeting, March 1994, Full paper is in Ichikawa's Laboratory [4] Instruction on CELL.FORT(2-DIM), Ichikawa's Laboratory, Dept. of Geotechnical Engineering, Nagoya University, February 1994 [5] A.M. Farahat(1992), Development of concrete models based on the micromechanics of granular material, Dr.Eng. Dissertation, Dept. of Civil Engineering, Nagoya University