III-171

A FEM simulation of strain localization in sand

Mohammed S. A. Siddiquee[1], T. Tanaka[2] and F. Tatsuoka[3]

Introduction:

The modeling of strain-softening in soils in the context of FEM is still a challenging research topic. Many sophisticated modeling of strain-localization have been attempted. In this paper, it is shown that very simple, experiment-based method can model shear localization quite close to the theoretical solution as well as experimental observation.

Modeling of strain-softening:

The solutions of bounadry value problems involving strain-softening material property are full of serious difficulties from both modeling of strain-localization numerical and mathematical points of view. The problem involves the spurious mesh dependency, which is an outcome of the loss of ellipticity of the incremental equilibrium equation [6] and the appearance of multiple equilibrium paths[6]. Several techniques have been proposed to bypass these unacceptable mesh-dependency. For the plasticity models the major available methods are:

1. The use of mesh size-dependent constitutive relations so as to obtain mesh-independent objective solutions[5,8] (as used in this study).
2. Nonlocal hardening[1]. These are basically an extension of the above model no.1. The internal plastic variable is averaged over a representative volume.
3. Gradient plasticity approach[2], including higher order derivatives in the strain or stress measurements.
4. Viscoplastic approach[4]. The main advantage is that by adding artificial viscous force it can retain the ellipticity of the incremental equilibrium equations. It uses the local bifurcation analysis, which shows the trigger of localization.
5. The micro-polar (Cosserat) continuum model. It is based on such an idea as that a macro-structure is subdivided into micro elements[3]. It adds an extra rotational degrees of freedom to the system.

A modified model-1 is used in this study to model strain softening as it is simple and practical to apply in boundary value problems such as the bearing capacity of footing on sand[7]. Unlike the original model-1 by Pietruszcak and Mroz[5], the modified one is not including the direction of shear band in the constitutive relation[8]. As it scales the plastic strain with a ratio of shear band width to the equivalent side-length of square of a FEM element, it is a non-local nature model in a limited sense. The gradient plasticity model and Cosserat model involve more terms in the incremental nonlinear equations due to the inclusion of higher order terms. The viscoplasic approach needs the cumbersome local bifurcation analysis throughout each analysis.

Description of the model used:

The additive decomposition of total strain increment isapplied while introducing a localization parameter in such a way that it takes into account the mesh size-sensitivity by including the area ratio of the actual to FEM localized zone, which in turn controls the rate of softening, as follows;

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + S d\varepsilon^{p}_{ij} \qquad (1)$$

where, $S = F_b / F_e = W / \sqrt{F_e}$, F_b is the area of the shear band in one element, F_e is the area of the element, W is the width of the shear band. Here, the shear band is considered as an intrinsic material property, although it is controversial whether it is a geometric property or material one. From the observation of plane strain compression (PSC) tests on sand, it has been found that at or near the peak condition, a shear band starts appearing and its width is 16 to 20 times the mean particle diameter [9]

The incremental nonlinear elasto-plastic stress-strain equation to be solved is given by:

$$d\sigma_{ij} = \left[D^{e}_{ijkl} - \frac{S D^{e}_{ijkl} b_{kl} a^{T}_{ikl} D^{e}_{klmn} d\varepsilon_{mn}}{A + S a^{T}_{ij} D^{e}_{ijkl} b_{kl}} \right] d\varepsilon_{ij} \qquad (2)$$

where D^{e}_{ijkl} is the elastic material property tensor, a_{ij} and b_{ij} are the stress-derivatives of yield function and plastic potential, respectively, A is the hardening-softening modulus, and $d\varepsilon_{ij}$ and $d\sigma_{ij}$ are the strain and stress increments.

FEM model simulation:

In order to explore the capabilities of this strain-softening model, the numerical PSC element test was simulated. The dimension of the specimen of 10 cm by 20 cm was modeled by 800 four noded quadrilaterals, and one point integration was used (Fig.1). The confining pressure applied was 0.8 kgf/cm^2. Vertical loading is modelled by applying uniform deformation from the perfectly lubricated top and bottom. Anti-hourglass scheme was used to suppress a possible zero-energy mode. The tolerance used was 10^{-2} of the initial force norm. All the numerical calculation were carried out in double precision to reduce the numerical inhomogeneity which may be introduced. In the analysis, no physical or numerical inhmogeneity (or imperfection) was intentionally introduced.

Other very important features of the model in triggering the localized mode of deformation are the non-associated flow rule coupled with Rowe's

[1] Graduate student, The University of Tokyo.
[2] Professor, Meiji University.
[3] Professor, The University of Tokyo.

$\varepsilon_a = 7.5\%$ $(\varepsilon_a)_f = 2.2\%$ $\varphi_{peak} = 47.17^0$ $\varphi_{mob} = 34^0$

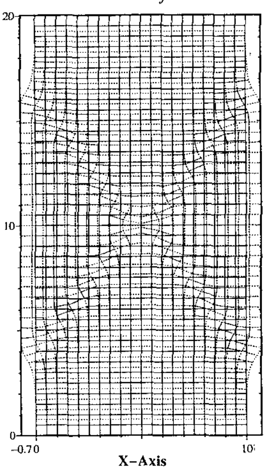

Fig.1 Deformed mesh

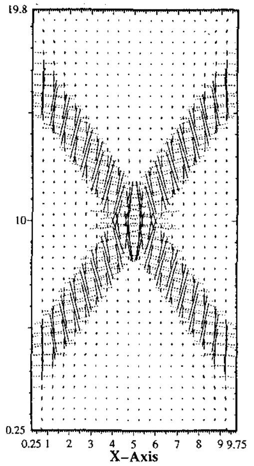

Fig.2 Vectors of principal strains

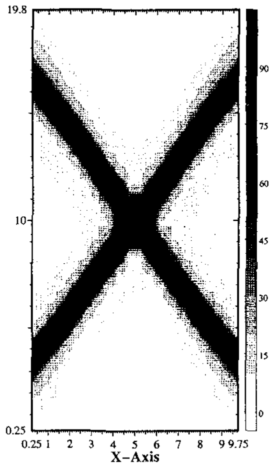

Fig. 3 Maximum shear strain contour

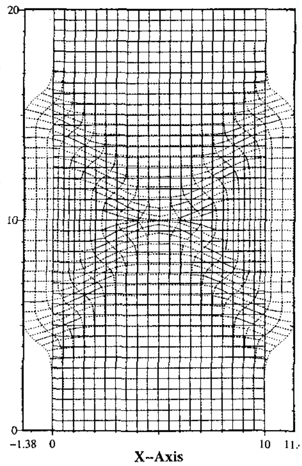

Fig. 4 Deformed mesh at large axial strain.

stress-dilatancy relation, pressure sensitivity, strength anisotropy [10].

Results and discussion:

Fig.1 through 3 shows the development of shear strains (plotted in the form of deformed mesh, the vectors of principal strains and the contours of max. shear strain) just after the peak (the average axial strain, ε_a=7.5 % versus the axial strain at peak= 2.2 %, and the mobilized friction φ_{mob}=34^0(residual) versus φ_{peak}= 47.17^0). It is seen that the theoretical perfect plasticity solution of shear band, i.e., the Roscoe surface at an angle of $\pi/4+\psi/2$ is exactly simulated. It is very interesting to note that shear banding is achieved without the introduction of any kind of inhomogeneity in the material. Most probably, the progressiveness of the solution (i.e., the dynamic relaxation method) itself is the reason for the localization to initiate from the center. Fig. 4 shows the deformed mesh at a later stage with ε_a=15 % and φ_{mob}=34^0 (residual).

There is a limitation inherent to this method. Namely, though the width of the shear band obtained is about 1.0 cm wide, it is much larger than the given width, 0.3 cm(i.e., smeared shear band). The main cause is in the inability of the four noded quadrilateral to form sharp gradients of deformation. This could bring some difficulty when analysing a boundary value problem in which the effect of boundary kinematic restraint on the shear banding is very large.

Conclusion:

A simple, practical method is used to model strain-softening in a PSC test on sand. The results obtained are close to the theoretical solution. This method can easily be used with a relevant existing FEM code for the analysis of bearing capacity problems.

References

1. Bazant, Z.P., and Feng-bao Lin, "Non-local yield limit degradation", Int. J. Num. Meth. Engng., 1988, **26**, 1805-1823.
2. Borst, R. de and Muhlhaus, H.-B., "Gradient-dependent plasticity: Formulation and algorithmic aspects", Int. J. Num. Meth. Engng., 1992, to appear.
3. Muhlhaus, H.-B. and Vardoulakis, I., "The thickness of shearband in granular materials", Geotechnique, 1987, **37**, 271-283.
4. Needleman, "Material Rate Dependence and Mesh Sensitivity in Localization Problems", Comp. Meth. Appl. Mech. Engng., 1988, **67**, 61-85.
5. Pietruszcak, S. T., and Z. Mroz, "Finite element of strain softening materials", Int. J. Num. Meth, Engng., 1981,**10**, 327-334.
6. Rudnicki, J.W. and Rice, J.R.,"Conditions for the Localization of Deformation in Pressure-sensitive Dilatant Materials", J. Mech. Phys. Solids, 1975, Vol. 23, pp. 371 to 394.
7. Siddiquee, M.S.A., Tanaka, T., and Tatsuoka, F., "A FEM simulation of model footing test on sand", *Proc. of 26th Japan National Conference on SMFE*, 1991, Nagano, pp. 1309-1312.
8. Tanaka and O. Kawamoto, 'Three dimensional finite element collapse analysis for foundations and slopes using dynamic relaxation', in *Proceedings of Numerical methods in geomechanics*, Insbruch, pp. 1213-1218 (1988).
9. Tatsuoka, F., Nakamura, S., Huang, C. C., and Tani, K., 'Strength anisotropy and shear band direction in plane strain tests of sand', Soils and Foundations, Vol. 30, No. 1, pp. 35-56 (1990).
10. Tatsuoka, M. S. A. Siddiquee, C.-S. Park, M. Sakamoto and F. Abe, 'Modeling stress-strain relations of sand', Soils and Foundations, Vol. 33, No.2, pp. 60-81 (1993).