III-158 Stress dependency in non-linear elastic behavior of soil

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INTRODUCTION

The non-linear elastic model taken into account the stress dependency of the stiffness was formulated and presented. It is believed that the elastic modulus of soil is dependent on the mean effective stress, $p'=(\sigma_1+\sigma_2+\sigma_3)/3$, (Hardin et al.1972) as;

$$E_i = K\, p'^n \quad \text{------------------------} \quad (1)$$

However, this empirical relation fails in explaining the results obtained from the cross-hole and down-hole tests (Yan et al.1991) in which the individual stress is an important factor determining the value of the corresponding elastic modulus. Roesler (1979) and Yu and Richart (1984) found in their laboratory tests that the velocity of elastic wave propagated through soil media in a specific direction is dependent on the stresses *in* the corresponding direction. They proposed the following empirical relation;

$$E_i = K\, \sigma_v^{\,n}\, \sigma_p^{\,m}\, \sigma_s^{\,q} \quad \text{--------------} \quad (2)$$

Where σ_v is the stress in the direction of wave motion, σ_p is the stress in the direction of particle motion. Note that the stress perpendicular to the plane of wave propagation, σ_s, has no effect on the elastic modulus of soil, then, q=0.

MODEL FORMULATION

Rowe (1971) proposed an empirical equation based on the Hertz contact theory for the elastic spheres as;

$$\varepsilon^e_{ii} = C_i\left\{\frac{\sigma_i}{E}\right\}^{1-n} \quad \text{-------------------} \quad (3)$$

When spheres were subjected to three dimensional confining stresses, the incremental form of the elastic formulation can be expressed using Hooke's law as shown in Eqn.(4). Differentiating Eqn.(3) and substituting into Eqn.(4) yields the non-linear incremental elastic formulation as expressed in Eqn.(5).

$$d\varepsilon^e_i = d\varepsilon^e_{ii} - \nu_{ij} d\varepsilon^e_{jj} - \nu_{ik} d\varepsilon^e_{kk} \quad \text{----------} \quad (4)$$

$$d\varepsilon^e_i = \frac{1-n}{E^{1-n}}\left[C_i \frac{d\sigma_i}{\sigma_i^n} - \nu C_j \frac{d\sigma_j}{\sigma_j^n} - \nu C_k \frac{d\sigma_k}{\sigma_k^n}\right] \quad \text{--------} \quad (5)$$

Considering the triaxial stress condition where $d\sigma_j=d\sigma_k=0$, and using the definition of elastic Young's modulus as shown in Eqn.(2), then,

$$(E_{tan})_{ij} = \frac{1}{(1-n)C_i} E^{1-n}\, \sigma_i^n = K\, \sigma_i^n\, \sigma_j^m \quad \text{-----} \quad (6a)$$

Similar expression can be obtained by assuming triaxial stress condition in the i direction ($d\sigma_i=0$) and $d\sigma_j=d\sigma_k$ as;

$$(E_{tan})_{ji} = \frac{1}{(1-n)C_i} E^{1-n}\, \sigma_j^n = K\, \sigma_i^n\, \sigma_j^m \quad \text{-----} \quad (6b)$$

Substituting Eqn.(6a) and 6(b) into Eqn.(5), the incremental form for the non-linear elastic model can be obtained as following;

$$d\varepsilon^e_1 = \frac{1}{K}\left\{\frac{1}{\sigma_3^m}\frac{d\sigma_1}{\sigma_1^n} - \frac{\nu}{\sigma_1^n}\frac{d\sigma_2}{\sigma_2^m} - \frac{\nu}{\sigma_1^n}\frac{d\sigma_3}{\sigma_3^m}\right\} \quad \text{----} \quad (7a)$$

$$d\varepsilon^e_2 = \frac{1}{K}\left\{\frac{1}{\sigma_1^n}\frac{d\sigma_2}{\sigma_2^m} - \frac{\nu}{\sigma_3^m}\frac{d\sigma_1}{\sigma_1^n} - \frac{\nu}{\sigma_1^n}\frac{d\sigma_3}{\sigma_3^m}\right\} \quad \text{----} \quad (7b)$$

where K is a elastic constant defined at very small strain levels (ε_a<0.001%) and can be determined from laboratory tests (Hardin et al.1972, Tatsuoka et al.1992 and Tanizawa et al.1994). Note that σ_i and σ_j was substituted by σ_1 and σ_2, and ε_i and ε_j were substituted by ε_1 and ε_2, respectively. Eqn.7(a) and (b) can be integrated to obtain an analytical form of the non-linear elastic relationship. In case of axial loading test under triaxial condition where $d\sigma_2=d\sigma_3=0$, the analytical form for the elastic axial deformation is;

$$\varepsilon^e_1 = \int d\varepsilon^e_1 = \frac{1}{K}\int_{(\sigma_1)_i}^{\sigma_1} \frac{1}{\sigma_3^m}\frac{d\sigma_1}{\sigma_1^n} = \frac{1}{\sigma_3^m K(1-n)}\left\{\sigma_1^{1-n} - (\sigma_1^{1-n})_i\right\}$$

Where $(\sigma_1)_i$ and $(\sigma_3)_i$ are the stress conditions before starting of shearing. Since $\sigma_3 = \sigma_2 = (\sigma_3)_i$ = constant and

$$K = \frac{E_{max}}{(\sigma_1)_i^n\, \sigma_3^m}$$

Then, the total elastic axial stress-strain relationship can be expressed as;

$$\varepsilon^e_1 = \frac{(\sigma_1^n)_i}{E_{max}(1-n)}\left\{\sigma_1^{1-n} - (\sigma_1^{1-n})_i\right\} \quad \text{-------} \quad (8)$$

The elastic lateral stress-strain relationship can also be obtained in a similar manner.

Note that the original concept proposed by Roesler (1979) in Eqn.(2) is that there is no specific stress directions for parameters n and m. This is very difficult in formulating the constitutive equation, therefore, in the present study, n and m were fixed to σ_1 and σ_3 directions, respectively.

PARAMETER DETERMINATION

The main objective of the paper is to construct a non-linear elastic model for simulating the unloading behavior of soil during excavation. The parameters should be, therefore, determined from the unloading tests. The model parameters, m and n, can be determined by using the triaxial test apparatus using the following assumption.

1) The maximum (or initial) Young's modulus, E_{max}, is defined as the Young's modulus determined at very small strain levels and complied to Eqn.(2).

$$E_{max} = K\,(\sigma_1)_i^n\,(\sigma_3)_i^m \text{ ----------- } (9)$$

2) The tangent Young's modulus, E_{tan}, obtained during unloading can be expressed also by Eqn.(2);

$$E_{tan} = K\,\sigma_1^n\,\sigma_3^m \text{ ------------------- } (10)$$

3) The ratio between the E_{tan} to E_{max} is, then;

$$\frac{E_{tan}}{E_{max}} = \left\{\frac{\sigma_1}{(\sigma_1)_i}\right\}^n \left\{\frac{\sigma_3}{(\sigma_3)_i}\right\}^m \text{ ----------- } (11)$$

4) For axial unloading test in which $\sigma_3=(\sigma_3)_i$, Eqn.(11) becomes;

$$\left\{\frac{(E_{tan})_{13}}{E_{max}}\right\} = \left\{\frac{\sigma_1}{(\sigma_1)_i}\right\}^n$$

or

$$\log\left\{\frac{(E_{tan})_{13}}{E_{max}}\right\} = n\log\left\{\frac{\sigma_1}{(\sigma_1)_i}\right\} \text{ ----------- } (12)$$

5) For lateral unloading test in which $\sigma_1=(\sigma_1)_i$, Eqn.(11) becomes;

$$\log\left\{\frac{(E_{tan})_{31}}{E_{max}}\right\} = m\log\left\{\frac{\sigma_3}{(\sigma_3)_i}\right\} \text{ --------- } (13)$$

Eqn.(12) and (13) suggest linear relationships between the normalized Young's modulus and normalized stress in the logarithmic plot.

Fig.1(a) and (b) show the results of two unloading tests, i.e., axial and lateral, using Toyoura sand specimens. It is clear from the figures that linear relationships between the normalized Young's moduli and the normalized stresses are valid as suggested by the equation.

CONCLUSION

The parameters for the proposed non-linear elastic model can be determined by performing two types of unloading tests in triaxial apparatus; i.e., axial and lateral unloading tests.The elastic modulus at small strain levels was taken into account.

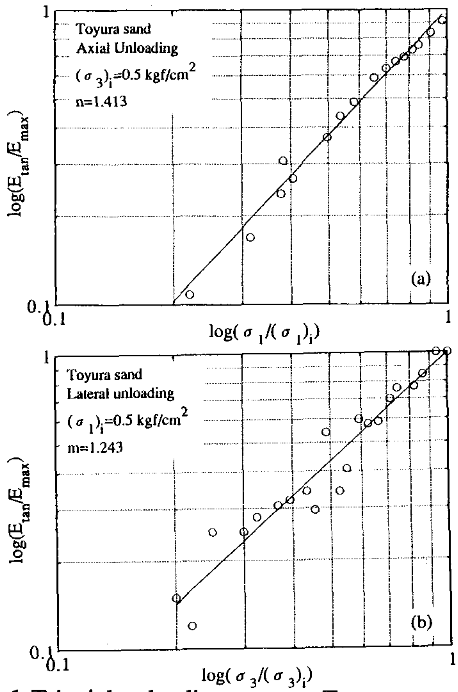

Fig.1 Triaxial unloading tests on Toyoura sand; (a) axial unloading test and (b) lateral unloading test

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