

III - 157 Young's modulus for elastic strains during triaxial compression of sands

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Introduction: It has been shown that the Young's modulus E^e of cohesionless soils for elastic normal strains in a certain direction is a rather unique function of the normal stress in that direction (Hoque et al. 1994). Corresponding to the above, the value of E^e for axial strains in very small unload/reload cycles during triaxial compression (TC) at a constant lateral stress increases (Kohata et al. 1994). On the other hand, it has been shown that the elastic shear modulus G^e decreases as approaching to the failure state in a torsional shear test and a torsional Resonant-Column test at a constant confining pressure (Tatsuoka, 1985, Yu and Richart, 1984). In the present study, the variation in E^e during TC bringing a sand specimen to failure was experimentally examined.

Testing procedure: In total four specimens of Toyoura and SLB sands with a height of 57cm and a 23cm * 23cm square cross-section were prepared by air-pluviation. LDTs (Local Deformation Transducer) were used to measure axial strains, ϵ_v , free from bedding error as well as lateral strains, ϵ_h , free from membrane penetration error (Hoque et al. 1994, Sato et al. 1994). Each specimen was first loaded isotropically ($K=1.0$) or anisotropically ($K=0.37$) upto $\sigma_h=7.0$ kgf/cm² and then rebounded to 0.8 kgf/cm². Subsequently the specimen was subjected to vertical and horizontal cyclic loading tests at several stress points along various stress paths including isotropic compression, $\Delta\sigma_v=0$, $\Delta\sigma_h=0$ and $\Delta\sigma_m$ (mean stress)=0 paths (Hoque et al. 1984). Finally, the specimen was sheared to failure by TC at $\sigma_h=0.8$ kgf/cm² while applying very small unload-reload cycles at several stress states to determine the elastic parameters (see Fig. 1). This paper describes the analysis of elastic parameters during TC.

Results and Discussion : Though the primary loading curves (as seen in Fig.1) may be noticeably affected by preloading, it is considered that the behaviour during small unload/reload cycles are not. Figs. 2 and 3 show two typical ϵ_v - σ_v relations in unload-reload cycles (at points A and B denoted in Fig. 1). Unloading and reloading parts at low shear stress (Fig. 2) are indistinguishable, whereas the difference is clear at high shear stress level (Fig. 3). The latter behaviour is due to the involvement of noticeable plastic strains. This phenomenon was observed more-or-less in all cyclic test data. Therefore, the slopes of unload/reload curves were obtained separately by linear regression for the same amplitude of vertical stress for unloading and reloading. Further, the true elastic Young's modulus E^e was obtained as illustrated in Fig. 4, namely;

$E^e = 2/(1/E_{\text{unload}} + 1/E_{\text{reload}})$ (1). This is obtained from the following Eqs. (2)~(4). For an axial stress change $\Delta\sigma_v$, the elastic axial strain amplitude is: $\epsilon_v^e = \Delta\sigma_v / E^e$ (2)

When the same plastic axial strain increment $\Delta\epsilon_v^p$ is involved during unloading and reloading, the axial strain increments are obtained, respectively, as:

$$(\Delta\epsilon_v)_{\text{unload}} = \Delta\epsilon_v^e - \Delta\epsilon_v^p = \Delta\sigma_v / E_{\text{unload}} \quad (3), \quad (\Delta\epsilon_v)_{\text{reload}} = \Delta\epsilon_v^e + \Delta\epsilon_v^p = \Delta\sigma_v / E_{\text{reload}} \quad (4)$$

The values of E^e , E_{unload} and E_{reload} for each test were divided by the Young's modulus for elastic axial strains obtained from the empirical relation established based on the results of the small cyclic tests performed at a range of stress ratio $1/2 \leq \sigma_v/\sigma_h \leq 2.0$ (Hoque et al. 1994);

$E^e = "E^e \text{ at } (\sigma_v)_1" \cdot \{ \sigma_v / (\sigma_v)_1 \}^m$ (5). The ratios are plotted against the stress ratio σ_v/σ_h in Figs. 5. The power $m=0.49$ and 0.42 for Toyoura and SLB sands, and $(\sigma_v)_1=0.8$ kgf/cm² and 2.2 kgf/cm² for Fig. 5a and Fig. 5b, respectively. The following trends of behaviour may be noted:

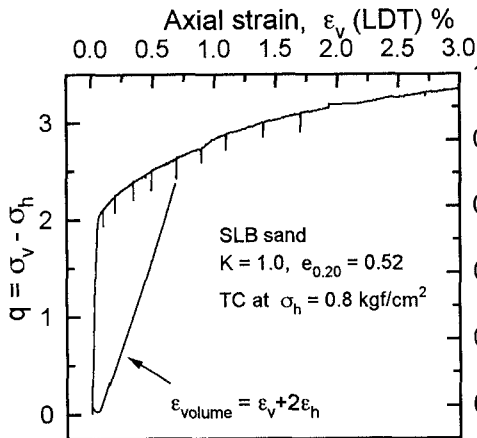
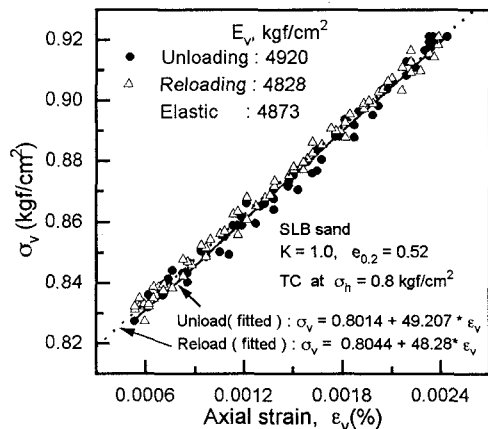


Fig. 1. Typical stress-strain relation from a TC test.

Fig. 2. Typical ϵ_v - σ_v relation at low shear stress.¹ Graduate student, The University of Tokyo.² Research Associate, The University of Tokyo.³ Professor, The University of Tokyo.

1) The difference between E_{unload} and E_{reload} becomes more noticeable as σ_v/σ_h increases probably due to involvement of larger creep in unload/reload cycles. This result suggests that the use of E_{unload} may overestimates the true Young's modulus E^e . 2) The Young's modulus E^e increases with σ_v/σ_h to its maximum value at a certain level of σ_v/σ_h , followed by a noticeable decrease. Namely, Eq.(5) overestimates the elastic Young's modulus E^e as approaching the peak stress state. This is probably due to such a microscopic change in the fabric as that the number of inter-particle contacts in the axial direction decreases drastically as approaching the failure state.

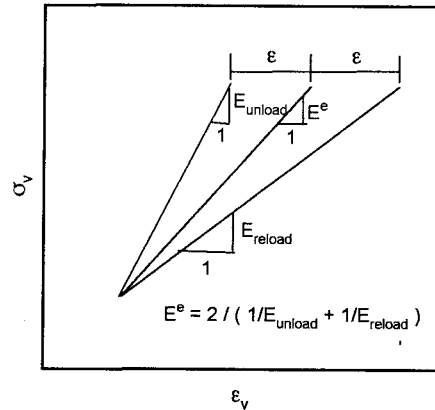
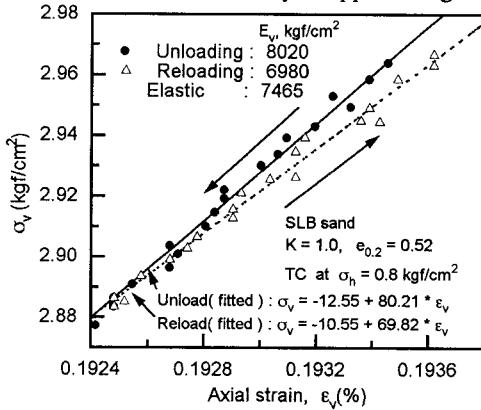


Fig.3. Typical ε_v - σ_v relation at high shear stress.

Fig.4. Definition of E_{unload} , E_{reload} and E^e .

Conclusion: In triaxial compression tests on sands, plastic strain increments involved in a small unload/reload cycle cannot be ignored as approaching the failure. The Young's modulus E^e for elastic axial strain increases with the increase in the axial stress, but as approaching the failure state, the rate of increase starts decreasing and finally the value of E^e starts decreasing.

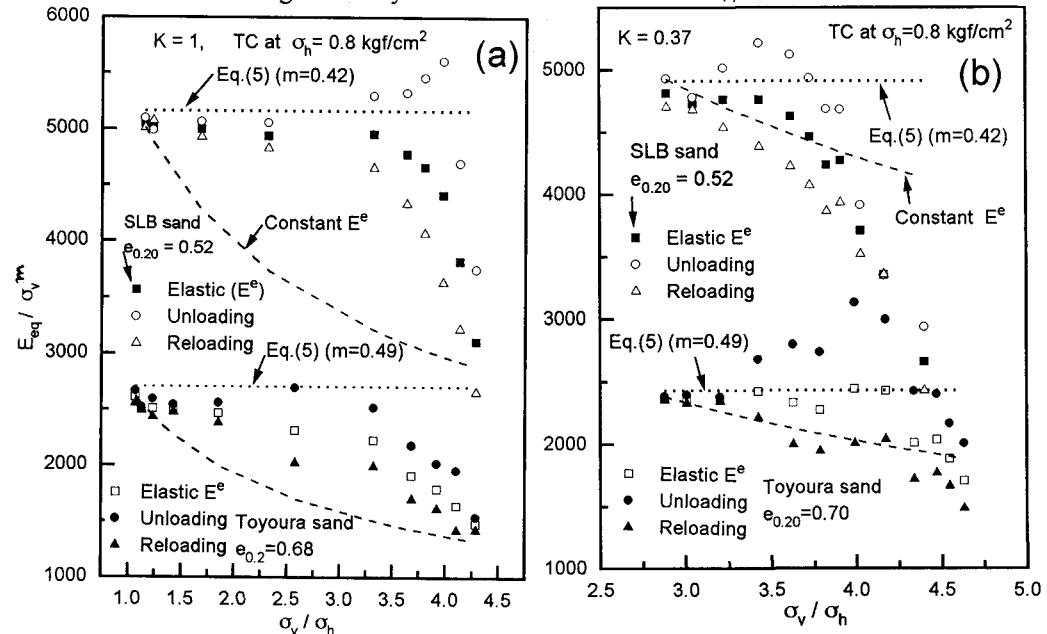


Fig.5. Variation of E_{eq} with σ_v/σ_h of Toyoura and SLB sands for a) $K=1$ and b) $K=0.37$ consolidation.

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