II-665 SIGNIFICANCE OF PARAMETERS IN THE DISPERSION MODEL AND ACCURACY OF THE FINITE ELEMENT METHOD WITH LINEAR INTERPOLATION FUNCTION

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Introduction Nowadays, mathematical models are widely used to study distribution patterns of the discharged pollutants in the receiving water body. The principle of mass conservation is the basis of these dispersion models. The complexity of the models depends on the involved water quality constituents which may vary from a single constituent, such as chloride, to a set of interrelated constituents, such as BOD-DO, nitrogen compounds, etc. Furthermore, the models can be classified according to their spatial dimensionality, time-scale condition, numerical techniques used in model formulation, and randomness or uncertainty of model parameters.

For successful use of the models in water quality management, clear understanding on the significance of model parameters is necessary. Besides the hydrodynamic phenomena of the water body, dispersion and reaction of the water quality constituents also play an important role in pollutant dispersion.

In the past few decades, the finite element method has been widely used in model formulation. It is a very powerful numerical technique which has been applied to solve various engineering problems including hydrodynamic circulation and pollutant dispersion in a water body. However, in some cases, there exist some oscillation in the results as well as problems related to model stability.

The objective of this paper is to discuss about the significance of some parameters in the dispersion model and the reliability of the dispersion model formulated by the finite element method with a linear triangular element.

Conservation of Mass Equation The conservation of mass equation can be written in the vectorial form as (Mauersberger, 1983):

$$\frac{\partial C}{\partial t} + \nabla.(vC - D.\nabla C) = G$$

where C is the substance concentration, v is the velocity vector, D is the diffusion coefficient tensor, and G is the source-sink term.

For a shallow water body, a vertically averaged two-dimensional mass balance equation can be derived by averaging over the depth. This equation can be written in the x-y coordinates as:

$$\frac{\partial c_i}{\partial t} + \frac{\partial (uc_i)}{\partial x} + \frac{\partial (vc_i)}{\partial y} - \frac{1}{h}\{\frac{\partial}{\partial x}(hD_x)\frac{\partial c_i}{\partial x} + \frac{\partial}{\partial y}(hD_y\frac{\partial c_i}{\partial y})\} + r_i c_i - R_i = 0$$

where c_i is the vertically averaged concentration, u and v are flow velocities, h is water depth, D_x and D_y are diffusion coefficients, r_i is the substance decaying rate, R_i is the source-sink term.

In this study, the Galerkin's weighted residual method is used in model formulation.

Significance of Model Parameters In this study, the Galerkin's weighted residual method is used in model formulation. The developed two-dimensional finite element model is applied to simulate dispersion pattern in a uniform channel. The rectangular channel, with 30 km in length, 100 m in width and 2 m in depth, is divided into 120 elements with a total of 93 nodal points. Two cases of pollutant dispersion are considered; 1) the substance concentration is specified at the upstream boundary, and 2) the pollutant loading is specified at a fixed section in the channel. In both cases, only the steady state condition is considered. The model parameters included in the analysis are dispersion coefficient, substance decaying rate, and flow velocity. Three values of the dispersion coefficient are considered, i.e. 1, 10 and 100 m^2/sec. while the substance decaying rates of 0.1, 1.0 and 10 per day are studied. The dispersion patterns are simulated at two different flow velocities, i.e. 0.05 and 0.5 m/sec.

In the case that substance concentration at the upstream boundary is specified, the ratios between the concentrations at the downstream nodes and the specified upstream concentration are plotted against the distance as shown in Figures 1 and 2. In the case that the pollutant loading is specified at a fixed section, the continuous rate of 2.0 g/sec is applied at the middle reach of the channel. The obtained results are shown in Figures 3 and 4.

The results obtained from the model simulation show that at high flow velocities, the value of dispersion coefficient has less significant effect on the pollutant dispersion pattern, while the value of substance decaying rate has a significant effect. However, the value of dispersion coefficient seems to have a remarkable effect at high substance decaying rate rather than at low decaying rate. The value of flow velocity is found to have an important effect on pollutant dispersion.

Accuracy of the Finite Element Model It is found that the developed finite element model using the linear triangular element can provide satisfactory results for the case that the substance concentration is specified at the upstream boundary. However, for the case that the pollutant loading is specified at a fixed section, oscillation in the results is observed at the upstream nodal points, especially when the value of dispersion coefficient is low. In this case, an element with higher-order interpolation function is recommended.

References

Connor, J.J. and C.A. Brebbia (1976),Finite Element Techniques for Fluid Flow, Butterworth & Co., London.

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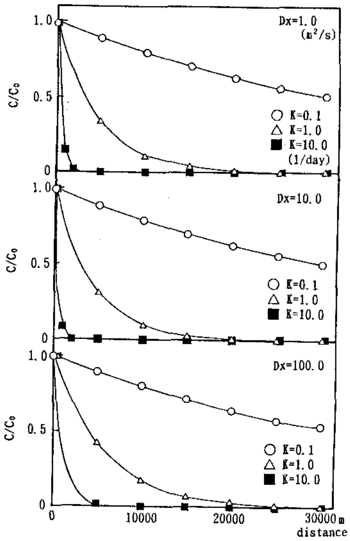

Figure. 1
Dispersion of pollutant in a uniform channel with specified concentration at the upstream boundary.
(flow velocity 0.05m/s)

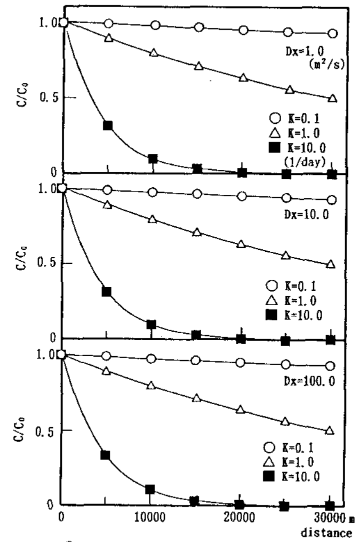

Figure. 2
Dispersion of pollutant in a uniform channel with specified concentration at the upstream boundary.
(flow velocity 0.5m/s)

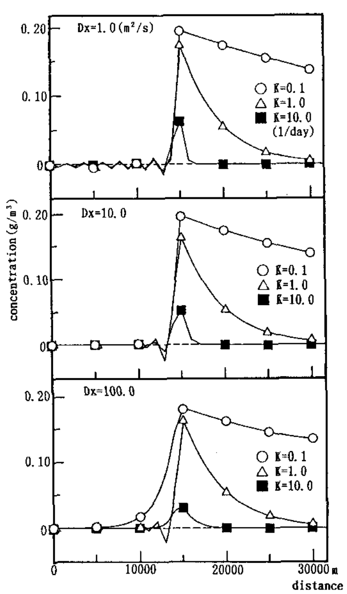

Figure. 3
Dispersion of pollutant from a single point source in a uniform channel with flow velocity of 0.05m/s

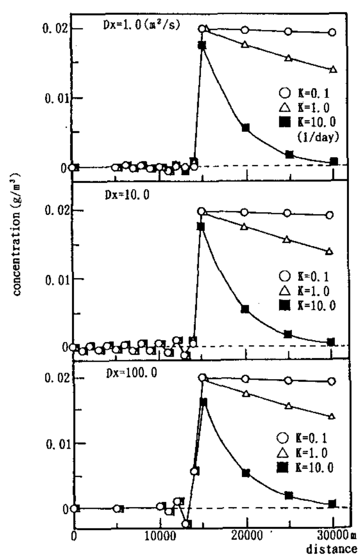

Figure. 4
Dispersion of pollutant from a single point source in a uniform channel with flow velocity of 0.5m/s