II-238 k-∈ TURBULENCE MODEL TO PLANE BUOYANT SURFACE JET

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1. Introduction

In the k−ε turbulence modelling, the representation of the buoyancy production term in the ε−equation has been left unresolved. Various modifications to the ε−equation have been suggested to improve the performance. For example, most of studies neglected the buoyancy term, while Rodi[1] claimed that it should be considered.

In this study, the numerical computations with the standard k−ε turbulence model were carried out in order to examine how the buoyancy production term must be treated in the ε−equation. The behaviors of two−dimensional surface buoyant jets with free surface boundary were investigated. Its computational results were compared with experimental results by Murota and Nakatsuji[2].

2. Governing Equation

Conservation equations of mass, momentum and buoyancy (or scalar quantities) are solved for an unsteady two−dimensional surface buoyant jet under the hydrostatic, boundary layer and Boussinesq approximation. The eddy viscosity ν_t can be determined from the turbulent kinetic energy k and the rate of dissipation ε as follows: $\nu_t=C_\mu \cdot k^2/\varepsilon$, in which C_μ is an empirical constant. The distributions of k and ε in the shear flows are obtained from modelled transport equations for these quantities, which are written as the following equations.

$$\frac{\partial k}{\partial t}+U_j\frac{\partial k}{\partial x_j}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)+\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}+\frac{\nu_t}{\sigma_t}\frac{\partial B}{\partial x_j}\delta_{3j}-\varepsilon \tag{1}$$

$$\frac{\partial \varepsilon}{\partial t}+U_j\frac{\partial \varepsilon}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right)+C_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}-C_{\varepsilon 2}\frac{\varepsilon^2}{k}+C_{\varepsilon 3}\frac{\varepsilon}{k}\frac{\nu_t}{\sigma_t}\frac{\partial B}{\partial x_j}\delta_{3i} \tag{2}$$

where σ_k, σ_ε, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, and $C_{\varepsilon 3}$ are the empirical constants. The values of these are used except $C_{\varepsilon 3}$ suggested by Launder and Spalding. For the constant $C_{\varepsilon 3}$, the buoyancy production term in the ε−equation, different values have been reported.

3. Computational Details

A control volume, finite difference procedure by Nakatsuji et al. is used to solve the above−mentioned unsteady governing equations. In order to avoid numerical instability, the free surface variations are connected together in implicit form, while other quantities are calculated by explicit two−step leap frog differencing in time derivatives and hybrid differencing in space derivatives of the convective term. The solution domain (400cm(x) X 120cm(z)) and boundaries adopted for the computations are corresponds to the experimental apparatus set by Murota and Nakatsuji[2].

4. Result and Discussion

The numerical computations are carried out in order to examine the effect of the buoyancy production term for the three cases, $C_{\varepsilon 3}$=0.0, 0.288 and 1.44, respectively. The value $C_{\varepsilon 3}$=0.0 is the case to neglect the buoyancy production term, the value $C_{\varepsilon 3}$=0.288 is recommended by Rodi and $C_{\varepsilon 3}$=1.44 is the case that the contribution of buoyancy production term is the same as that of the shear production term. Figure 1 shows the longitudinal variations of vertical profiles of the computed buoyancy with the experimental results. As shown in the figure, the effect of the buoyancy production term appears apparently in the case of Fdo=3.3. It can be seen that the consideration of the buoyancy production term would improve the level of the stratification. As shown in figures, the computed result $C_{\varepsilon 3}$=1.44 has a good agreement with the experimental data. Figure 2 shows longitudinal variations of vertical profile of the computed velocities using $C_{\varepsilon 3}$=1.44 for Fdo=3.3, 6.0 and 9.0 with the experimental results. The computational results are in a good agreement with the experimental data.

The vertical distributions of computed eddy viscosities ν_t are presented in Fig. 3 corrresponding to Fig. 2. It is well represented that the values of ν_t tends to become small with decreasing Fdo. When the constant value of $C_{\varepsilon 3}$ is set to 0.0, the spreading rate becomes the same value independent of the value of Fdo. With an increase of $C_{\varepsilon 3}$, based on Eqs.(1) to (2), the dissipation rate ε increases so that the turbulent kinetic energy k decreases. Such an effect cannot be appeared in the computation with $C_{\varepsilon 3}$=0.0. It can be concluded that the stratification effects can be introduced reasonably by use of the buoyancy production term in the k−ε turbulence modelling.

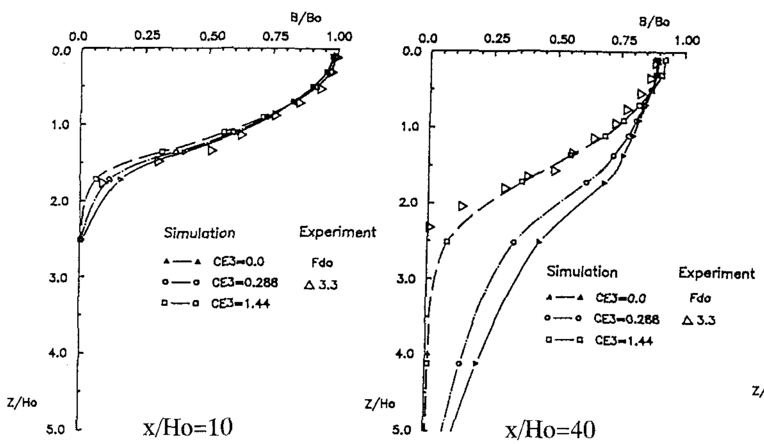

Fig. 1 Effects of $C_{\varepsilon 3}$ values

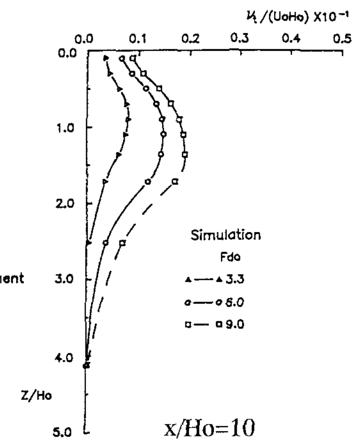

Fig. 3(a)

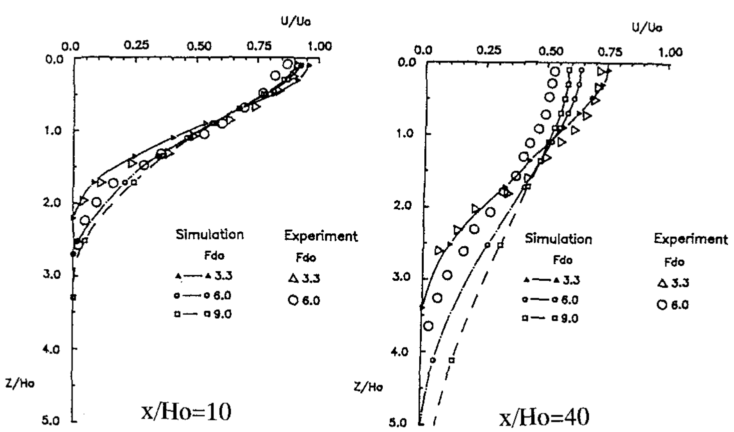

Fig. 2 Comparison with Experimental data against Fdo

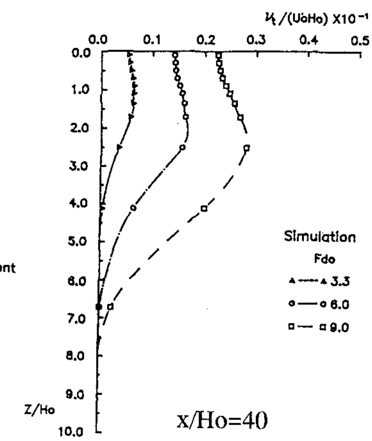

Fig. 3(b)

Computed eddy viscosities against Fdo

References

[1] Rodi, W. : Proc. 2nd Sympo., Turbulent Shear Flows, 10, 37(1979).
[2] Murota, A. & Nakatsuji, K. : Proc. JSCE, 352/II-2, 97(1984).