II-66 Development of Stochastic Rainfall Model Associated with Monthly Air Temperature

Michio Hashino*, Sheng Yue** and Youko Nagura**
* Professor and ** Graduate Students, The University of Tokushima

1. Introduction

Using the data of monthly mean air temperature and daily rainfall amount during the past one hundred years in Tokushima, authors try to develop stochastic models utilized to estimate the changes of one storm properties associated with fluctuations in monthly air temperature.

2. Stochastic Daily Rainfall Model

One storm cluster as shown in Fig. 1, is defined as a continuous daily rainfall. Continuous rainy days T_r, continuous dry days T_b, the days between two adjacent storm T_a and the total rainfall amount of one storm cluster R are assumed as the properties of one storm cluster.

For the distribution of number of storm cluster N_t, according to the relationship between mean and variance of N_t, it can be known that, of the binomial distribution, Poisson distribution, negative binomial distribution, which one is the most suitable one for N_t[1]. As the mean of N_t : $E(N_t)$ is given as Eqs. (1), (2) and (3), then the variance of N_t is obtained by Eq. (2)[2]. In this case, the mean of T_a: $E(T_a)$ can be easily obtained by convolution integral of bivariate Freund distribution[3] on the relation between T_r and T_b.

For the distribution of one storm amount R, as it follows a gamma distribution with three parameters, then mean $E(R_j)$, variance $V(R_j)$ and the coefficient of skewness $C(R_j)$ are shown as Eq. (5) with three parameters.

3. Annual Fluctuation Models of $E(N_t)$ and E(R) Influenced by Monthly Air Temperature

For builting up the annual fluctuation models of $E(N_t)$ and E(R), considering that $E(N_t)$and E(R) respectively have some relation with monthly mean air temperature, then $E(N_t)$and E(R) can be given as Eqs. (6) and (7) represented by the product of two components with monthly air temperature ingredient and sinusoidal function.

For the method to estimate the parameters in the models, strictly, because the observation value of the mean in the left of equation unexists, maximum likelihood method has to be utilized to estimate the parameters. But, here simply, the mean of observation data is considered as the mean in the left of equation. Then according to the step-

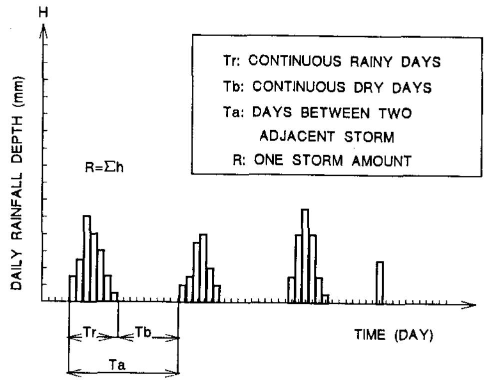

Fig. 1 Schematic Definition of Properties of One Storm Cluster

$$E(N_t)=D_j/E(T_a)=D_j\exp(\varepsilon_j t+\xi_j) \quad (1)$$

$$V(N_t)=E(N^2{}_t)-E(N_t)^2 = E(N_t)^2(\varepsilon_j t-1)\exp(-\varepsilon_j t)+E(N_t) \quad (2)$$

$$f(T_r,T_b)=\begin{cases} a_1 b_2\exp[-b_2T_b-(a_1+b_1-b_2)T_r] & (0<T_r<T_b) \\ b_1 a_2\exp[-a_2T_r-(a_1+b_1-a_2)T_b] & (0<T_b<T_r) \end{cases} \quad (3)$$

where D_j:one month period;
t :time variable;
$\varepsilon_j, \xi_j, a_1, b_1, a_2, b_2$:parameters.

$$f(R_j)=\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}(R_{ji}-r_j)^{\alpha_j-1}\cdot\exp[-\beta_j(R_{ji}-r_j)] \quad (4)$$

$$E(R_j)=\frac{\alpha_j}{\beta_j}+\gamma_j,\quad V(R_j)=\frac{\alpha_j}{\beta^2_j},\quad C(R_j)=\frac{2}{\sqrt{\alpha_j}} \quad (5)$$

where R_j:total rainfall month j;
α_j, β_j, r_j:parameters.

$$E(N_{tji})=D_j\exp[C_{j0}+C_{j1}\theta_{ji}+C_{j2}\theta^2{}_{ji}+C_{j3}\theta^3{}_{ji}]\cdot\exp[\sum_{i=1}^{lp}k_{ji}\sin\{2\pi(i+\phi_{ji})/\omega_{ji}+d_{ji}\}] \quad (6)$$

$$E(R_{ji})=\frac{\alpha_j}{\beta_j}+\exp[a_{j0}+a_{j1}\theta_{ji}+a_{j2}\theta^2{}_{ji}+a_{j3}\theta^3{}_{ji}]\cdot\exp[\sum_{n=1}^{np}k_{jn}\sin\{2\pi(i+\phi_{jn})/\omega_{jn}\}] \quad (7)$$

where θ_{ji}:the monthly mean air temperature;
ϕ_j:phase;
ω_j:cycle in month j;
c_j, a_j, k_j, d_j:parameters.

wise regression, the multiple regression analysis is carried out to estimated the parameters in Eqs. (6)and(7). Also, AIC criterion is used as a criterion of parsimony of parameters.

4. Application Result

For the distribution of number of storm cluster N_t, it is found that in Tokushima it follows binomial distribution(mean>variance) as shown in Fig. 2, where Poisson distribution(mean=variance) is also shown.

For the distribution of one storm amount R, Gringorton formula is utilized to draw a graph in log-probability paper as shown in Fig. 3, so that gamma distributions are well suitable for R.

The annual fluctuations of $E(N_t)$ and E(R) are respectively calculated by utilizing Eqs.(6) and (7). For the sake of comparison, the models only influenced by monthly air temperature are used to estimate $E(N_t)$ and E(R). From Figs. (4) and (5), it can be seen that $E(N_t)$ and E(R) are still influenced by other time series ingredients besides monthly air temperature.

5. Afterword

From Figs.4 and 5, it can be seen that, qualitatively, the models can explain the observation data well. But, quantitatively they can not still explain the data well. So, in the future, the most important thing is that the parameters in models have to be re-estimated by utilizing maximum likelihood method.

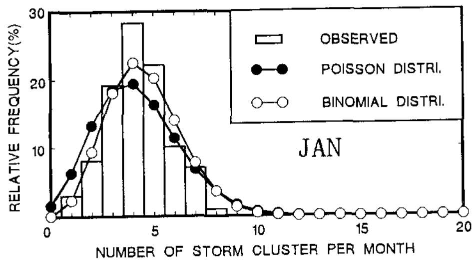

Fig. 2 Relative Frequency of One Storm Cluster(January)

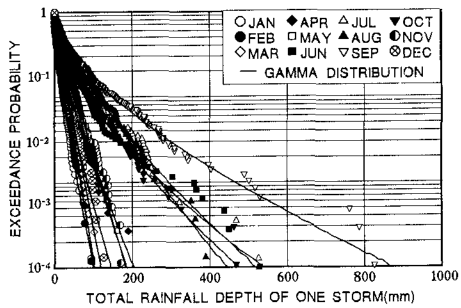

Fig. 3 Probability Distribution of Total Rainfall of One Storm Cluster(Jan.)

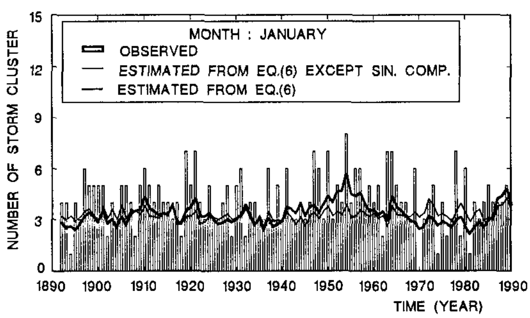

Fig. 4 Annual Fluctuation of the Mean of Number of Storm Cluster(Jan.)

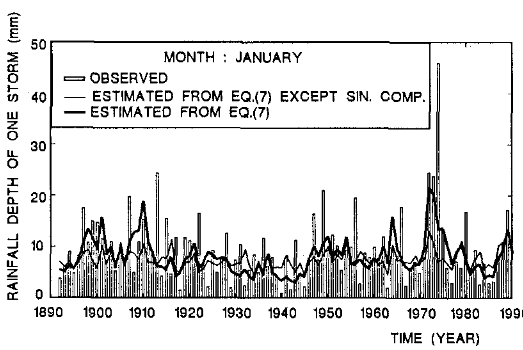

Fig. 5 Annual Fluctuation of the Mean of Total Rainfall of One Storm Cluster (January)

References

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2) Snyder, D.L.: Random Point Process, John Wiley & Sons, 1975.
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