II－2 Aquifer Parameter Estimation and Optimal Allocation of Pumping Wells

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1. Introduction. Inverse analysis of determining the model parameters based on the observation data of variables on space and time has become widely used in geotechnical engineering, including quantitative groundwater hydrology. Different criteria have been used in this work to estimate the parameters of the confined aquifer of Hanoi city, Vietnam.

The optimal allocation of pumping wells is an important aspect of the management of groundwater basin. An approach of searching the optimal location of a number of pumping wells that should satisfy some constraints has been proposed in this paper and has been applied for Maidich pumping field in the western part of this city. The city's area where the pumping wells are mainly distributed is about 400 km^2.

2. Methods.

Inverse Analysis. One of the methods used in the inverse analysis is the method to estimate the parameters by minimizing the objective function ($J(\underline{\theta}|\lambda)$) consisting both of observation data ($J_h(\underline{\theta})$) and prior information ($J_p(\underline{\theta})$) of the parameters[1]:

$$J(\underline{\theta}\,|\,\lambda)=J_h(\underline{\theta})+\lambda J_p(\underline{\theta}) \text{ where } J_h(\underline{\theta})=[\underline{h}^*-\underline{h}]V_h^{-1}[\underline{h}^*-\underline{h}],\ J_h(\underline{\theta})=[\underline{\theta}^*-\underline{\theta}]V_\theta^{-1}[\underline{\theta}^*-\underline{\theta}] \quad (1)$$

where: λ: a positive scaler adjusting the relative strength of the subjective information to the objective information, $\underline{h}^*$, $\underline{h}$: the observed and calculated (model) head, respectively, $\underline{\theta}^*$, $\underline{\theta}$: the prior and model parameter, $\underline{\theta}^*$: the prior mean of the model parameter θ which is assumed to follow a multivariate normal distribution, V_h: the covariance matrix of the error vector, V_θ: the prior covariance matrix of parameter θ.

The second term $J_\theta(\underline{\theta})$ is employed to stabilize the estimation. This method is termed Extended Bayesian method[2]. Thus for different values of λ there are existing different model parameters, e.g., models. On the other hand, it has been well-known that the Maximum Likelihood Method (MLM) is a method to choose the most appropriate model parameters, which have the maximum expected average log-likelihood, among several given models. The expected average log-likelihood is:

$$E[\ln\{f_k(x\,|\,\theta_k(x)\}]=\int f(x)\ln\{f_k(x\,|\,\theta_k(x)\}dx \quad (2)$$

Furthermore it has been proved that the unbiased estimator of the expected average log-likelihood is given by Akaike Information Criterion (AIC)[3]:

AIC=-2x(the maximum log-likekohood)+2x(number of free model parameter) (3)

The best model is that whose parameters minimize AIC.

The procedure to carry out the inverse analysis here is that different values of hyper-parameter λ are inputted, the model parameters are calculated by minimizing $J(\underline{\theta}|\lambda)$ in Eq. (1), AIC is determined and the minimal AIC corresponds to the best model.

Optimal Allocation of Pumping Wells. This problem can be stated as follows. In a particular groundwater pumping field it is required to allocate N pumping wells, the pumping rates of which are given as functions of space (x, y). The constraints are that the water levels at the wells should not be lower than the given levels which are given as functions of space (x, y) and time (t) and the total distance from them to the given water treatment plant (termed as the center) is minimal.

By means of Cooper-Jacob approximation solution[4] for the case of group-wells, the water level drop, e.g., the difference between the initial level H_j and disturbed level h_j at any pumping well j is:

$$H_j-h_j=\frac{Q_j}{2\pi KM}\log\frac{R}{r_{jj}}+\sum_{i=1,j\neq i}^{N}\frac{Q_i}{2\pi KM}\log\frac{R}{r_{ij}};\quad where\quad R=1.5\sqrt{\frac{KMt}{S}} \quad (4)$$

where: Q_i and Q_j: the pumping rate of well i and j, respectively, R: the influence radius of the wells[5], K: the permeability, M: the thickness, S: the storage coefficient, r_{ij}: the distance between j and i, r_{jj}: the radius of well j, t: the pumping time.

From Eq.(4) and the requirement of the water level constraint it follows that:

$$H_j - \left[\frac{Q_j}{2\pi KM} \log \frac{R}{r_{jj}} + \sum_{i=1,j\neq i}^{N} \frac{Q_i}{2\pi KM} \log \frac{R}{r_{ij}} \right] \geq (H_j)_{\min} \tag{5}$$

where: $(H_j)_{min}$: the minimal water level allowed at well j.

Suppose now that there are M (M>N) candidates-wells. We are going to choose only N from M that satisfy our target. Now let us choose M candidates-wells in a more systematical way. We divide the area into a number of rectangular meshes with equal spacing. The number of the grids is M. If the solution exists, then it should be one (or more than one) among M!/[N!x(M-N)!] combinations. The number of combinations may be very large, thus it requires some skill to choose the mesh with smaller number of grids, ensuring the solution exists and the solution is as much close to the true unknown one as possible.

3. Results and Conclusions. In the inverse analysis seven zones of different transmissivity and specific storage have been chosen. The initial transmissivity values of the zones are 500, 700, 900, 1100, 1300, 1500 and 1700 m^2/day. The specific storage S_s=0.0003/m for zone with T=500 m^2/day, S_s=0.0004/m for T=700-900 m^2/day, S_s=0.0005/m for T=1100-1300m^2/day and S_s=0. 0006/m for T=1500-1700 m^2/day.

The results of estimation of transmissivity with constant prior parameters, which are equal to the zones' values, for unsteady state are presented here. The observation water levels of 16 observation wells have been used. The time discritization in the calculation was 10 days and the observation intervals were 30 days corresponding to February, March and April 1991. The AIC minimize at λ=0.0004 (Figure 2) and the estimated parameters for the zones are 508, 858, 1299, 1692, 1740, 1657 and 2414m^2/day. The small value of λ indicates the insignificant role of the prior information.

The aquifer's schematic section is drawn in Figure 1. For a long term pumping the aquifer parameters used in Eq. (5) are the transmissivity of the confined aquifer and the storavity is those of the unconfined aquifer[5]. The time is 5 years (by this time the influence radius reaches the recharge boundary). The location of the water treatment plant and 18 present pumping wells is shown in Figure 3. The total number of new pumping wells is 17 with the planned pumping rate 4,200 m^3/day. Thus, the total water pumping increase is 71,400 m^3/day. The pumping field is divided into 5x6 grids with spacing 0.75 km. The optimal location of the wells is plotted in Figure 3 with the sum of the distances from each well to the water treatment plant equal to 30.742 km.

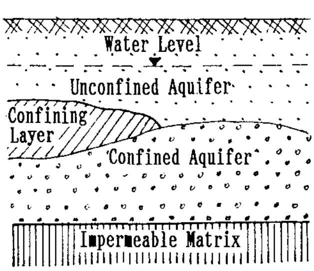

Figure 1. Schematic Section of the Aquifer

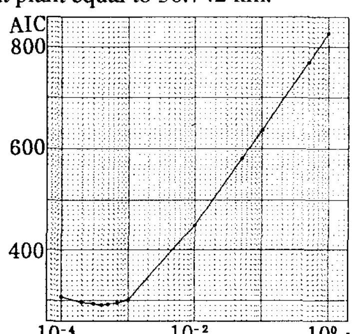

Figure 2. AIC versus Hyperparameter λ

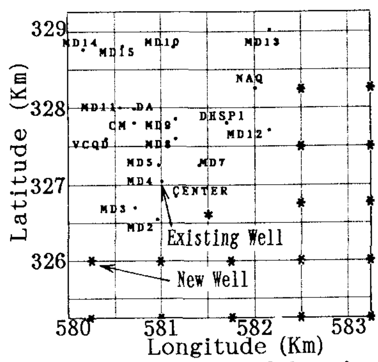

Figure 3. Location of Pumping Wells in Maidich Field

References

1. Neuman, S. P. & Yakowits, S., A Statistical Approach to the Inverse Problem of Aquifer Hydrology. Water Resources Research, Vol. 22, No. 2, Pp. 95-108.
2. Honjo, Y., (1991), A Statistical Procedure to Match Subjective and Objective Information in Geotechnical Inverse Analysis. Sixth International Conference on Application of Statistics and Probability in Civil Engineering. Mexico, June 1721, 1991.
3. Akaike, H., (1989), Bayesian Statistics and Its Applications, University of Tokyo Press, Tokyo. (The English translation by Dr. Honjo, Y.)
4. Bowen, R., (1986), Groundwater, Elsevier Applied Science Publishers, London.
5. Maximov, V. M., (1979), A Handbook of Hydrogeologist, Publishers Nedra, USSR. (in Russian)