I - 843

A SIMPLE PROCEDURE FOR DYNAMIC ANALYSIS OF CONTINUOUS BRIDGES

Wilson GORGES, Graduate Student, Miyazaki University
Takanori HARADA, Associate Professor, Miyazaki University
Takeshi NEMOTO, Graduate Student, Miyazaki University

1 Introduction

For bridges, there is still a need to improve our understandings of the dynamic soil-structure interaction effects. A simple and analytical approach for the evaluation of fundamental period and damping ratio of continuous bridges, in which the dynamic soil-foundation interaction effects are included, is presented in this paper.

2 Equivalent SDOF System of a Single-Support Bridge

A lumped-mass-spring model as shown in Fig. 1 is examined. The structure is modeled with a mass m_S supported by a massless column with complex dynamic stiffness K_S^*, function of the frequency ω. The foundation is embedded into soil, and modeled with a mass M and a mass moment of inertia J_G. The soil's dynamic stiffnesses are represented by the springs K_{hh}^*, K_{rr}^*, and K_{hr}^*. The effective input motions are represented by the swaying component u_{CT} and the rocking component θ_{CT}.

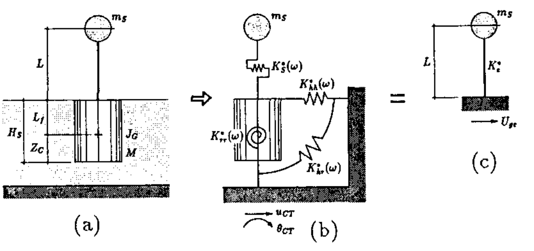

Fig. 1 Equivalent SDOF lumped-mass-spring model.

The equations of motion of this system can be written in frequency domain as

$$(-m_S\omega^2 + K_e^*)\, U_e = m_S\omega^2\, U_{ge} \tag{1}$$

where,

$$K_e^* = \frac{A\, K_S^*}{A + B\, K_S^*}, \quad U_{ge} = \frac{C}{A}u_{CT} + \frac{D}{A}L\theta_{CT}, \quad U_e = \frac{K_S^*}{K_e^*}u \tag{2}$$

and

$$\begin{aligned}
A &= MJ_G\omega^4 - [J_G K_{hh}^* + ML_f(L_f K_{hh}^* + 2K_{hr}^*) + MK_{rr}^*]\omega^2 + (K_{hh}^* K_{rr}^* - K_{hr}^{*2}) \\
B &= -[J_G + M(L+L_f)^2]\omega^2 + (K_{rr}^* + K_{hh}^* L^2 - 2K_{hr}^* L) \\
C &= -[J_G K_{hh}^* + M(L+L_f)(L_f K_{hh}^* + K_{hr}^*)]\omega^2 + (K_{hh}^* K_{rr}^* - K_{hr}^{*2}) \\
D &= -\Big[\frac{J_G K_{hr}^*}{L} + \frac{M(L+L_f)}{L}(L_f K_{hr}^* + K_{rr}^*)\Big]\omega^2 + (K_{hh}^* K_{rr}^* - K_{hr}^{*2})
\end{aligned}$$

The quantities A, B, C and D are only functions of soil-foundation properties. Equation (1) thus can be interpreted as the equation of motion of the equivalent SDOF system shown in Fig. 1c. All the effects of dynamic soil-foundation interaction are included in the equivalent complex spring K_e^* and the equivalent input motion U_{ge}, defined by Eq. (2).

3 Equivalent SDOF System of a Multiple-Support Bridge

The theory of the equivalent SDOF system of a single-support bridge can be extended to obtain an equivalent SDOF system of a multiple-support bridge, by employing the principle of virtual displacements. A continuous girder-pier-foundation system can be modeled into a continuous beam having distributed mass $m(x)$ and flexural rigidity $EI(x)$, which is supported by equivalent complex springs K_{ej}^* with fixed-base condition, and is subjected to equivalent input motions U_{gej}, as shown in Fig. 2.

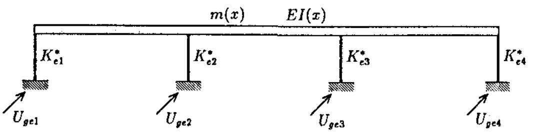

Fig. 2. A multiple-support continuous bridge model.

The shape function $\psi(x,\omega)$ of the continuous beam due to the loading $w(x) = m(x)g$ being applied laterally, either in longitudinal or transverse directions, as shown in Fig. 3 is then evaluated.

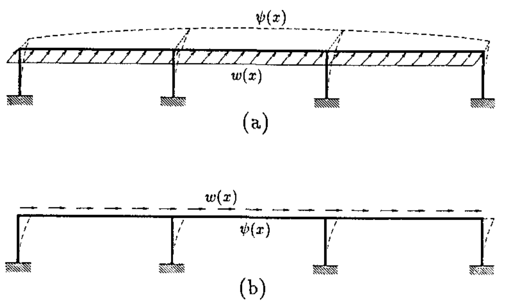

Fig. 3. Shape functions: (a) transverse direction; (b) longitudinal direction.

Using the virtual work method, the following equation of motion can be obtained [1]

$$[-\omega^2 m^* + i\omega c^* + k^*]\; z^t = [i\omega c^* + k^*]\; z_g^* \tag{3}$$

where m^* is the generalized mass, k^* is the generalized spring coefficient, c^* is the generalized damping coefficient, and z_g^* is the generalized input motion. Each is defined as follows:

$$m^* = \frac{1}{g}\int w(x)\psi^2(x)dx, \quad k^* = \int w(x)\psi(x)dx \tag{4a, b}$$

$$c^* = \frac{1}{\omega}\sum_j Im(K_{ej}^*)\psi^2(x_j), \quad z_g^* = \frac{\int w(x)\psi(x)\Delta_S dx}{\int w(x)\psi^2(x)dx} \tag{4c, d}$$

The natural period T and the damping ratio h can be found, respectively, as

$$T = \frac{2\pi}{\omega_0}, \quad h = \frac{c^*(\omega_0)}{2\sqrt{m^*(\omega_0)k^*(\omega_0)}} \tag{5}$$

where ω_0 is the frequency that gives the maximum value of the transfer function $|z^t/z_g^*|$. Since the complex spring K_e^* is function of frequency, as well the generalized mass, spring and damping coefficient, the natural period and the damping ratio given by Eq. (5) must be determined by iteration.

4 Numerical Example

A three-span highway continuous bridge model is examined [1]. The results obtained by using the formulation of the simple procedure presented here are compared with those obtained by the analysis of a MDOF system. All supports are subjected to the same input motion with unitary translational component u_{CT}. In order to illustrate the influence of the foundation size on the bridge response characteristics, the radius of the two inner foundations are changed as $a_2 = a_3 = 2$ m, 6 m and 10 m.

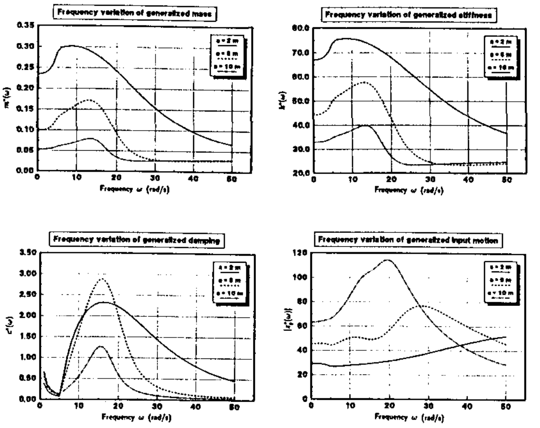

Fig. 5. Frequency variation of generalized mass, spring, damping coefficient, and input motion.

The frequency variation of the generalized mass $m^*(\omega)$, spring $k^*(\omega)$, damping coefficient $c^*(\omega)$, and input motion $z_g^*(\omega)$ are shown in Fig. 5. It is observed from Fig. 5 that the frequency variation is significant in the generalized mass, spring, and damping coefficient. In high frequency range, their values tend to approach constant values — which correspond to the values for fixed-base condition — independent of the foundation size.

The transfer function $|z^t/z_g^*|$ defined by Eq. (3) is shown in Fig. 6. The fundamental period and the damping ratio determined by Eq. (5) are given in the following table:

Case	$a_2 = a_3$	ω_0 (rad/s)	T (s)	h
1	2 m	15	0.42	0.25
2	6 m	32	0.20	0.15
3	10 m	30	0.21	0.06

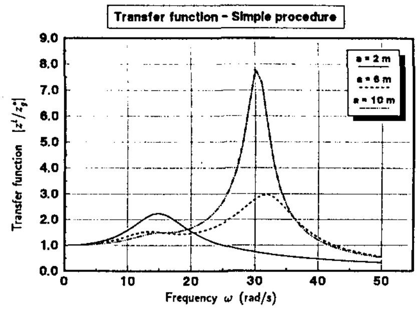

Fig. 6. Transverse frequency response transfer function obtained from the simple procedure.

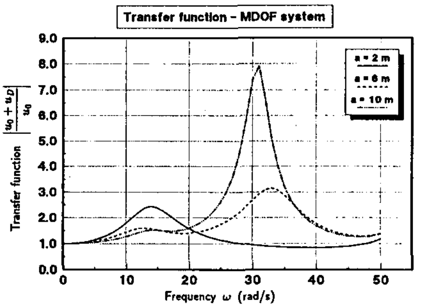

Fig. 7. Transverse frequency response transfer function obtained from the analysis of the MDOF system.

On the other hand, the frequency response transfer function is computed by using a MDOF system with 28 nodes. Figure 7 shows the frequency response function for the total response displacement $|u_0 + u_D|$ normalized by the pseudostatic displacement $|u_0|$ at the top of pier P2. Comparison of the computed response characteristics shown in Fig. 7 with those in Fig. 6 may indicate the validity of this simple procedure.

5 Conclusions

A simple procedure for the evaluation of the fundamental period and the damping ratio of continuous bridges has been presented. A concept of equivalent complex spring and equivalent input motion of a soil-foundation-pier system has been derived. This concept makes it possible to assess the dynamic soil-structure interaction effects on the seismic responses of bridges. Comparison of results obtained by the simple procedure with those obtained by the MDOF analysis may indicate the validity of the approach presented here.

6 References

[1] Harada, T., Sakanashi, K., and Gorges, W. "A Method for the Evaluation of the Period and the Damping Ratio of Continuous Girder-Pier-Foundation System". (in Japanese) *Journal of the JSCE*, April, 1994.