I - 514 ACTIVE CONTROL OF FLUTTER INSTABILITY OF BRIDGE DECK WITH RATIONAL FUNCTION APPROXIMATION OF AERODYNAMIC FORCES

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1. INTRODUCTION: The equations of motion of a flexible bridge subjected to wind contain unsteady generalized aerodynamic forces, which are functions of reduced frequency, are obtained from either analysis or experiment in the form of tabular data. Rational Function Approximations (RFA), namely the Minimum State RFA formulation (Karpel), allow the aeroelastic equations of motion to be cast in a linear time invariant state-space form. Utilizing the rational function approximation the application of optimal control theory to flutter of the bridge deck with additional control surfaces is studied.

2. RATIONAL FUNCTION APPROXIMATION TO GENERALIZED AERODYNAMIC FORCES: A bridge deck of width B submerged in a smooth flow is assumed to have two degrees of freedom: bending displacement and torsion, denoted by h and α respectively. The flutter equation of the deck is cast in the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{V}_q\mathbf{Q}\mathbf{q} \tag{1}$$

where $\mathbf{q} = [h/B \ \ \alpha]^T$ and $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are mass, damping and stiffness matrices, $\mathbf{V}_q = diag(-0.5\rho U^2 B^3, 0.5\rho U^2 B^2)$ and $\mathbf{Q}$ is a complex matrix built of flutter derivatives H_i^*, A_i^* $(i = 1,..,4)$ which are functions of $K = B\omega/U$. The most common form of the approximating function for each coefficient Q_{ij} used in aeronautics is a rational function of the nondimentional Laplace variable p $(p = s\,B/U = iK)$.

$$\hat{Q}_{ij}(p) = (A_0)_{ij} + (A_1)_{ij}\,p + \sum_{\ell=1}^{n_\ell}(A_{\ell+1})_{ij}\frac{1}{p+\lambda_\ell} \tag{2}$$

$\hat{Q}_{ij}$ is the reduced-frequency domain tabular data of force coefficients. The partial fractions are called lag terms since each represents a transfer function in which the output "lags" the input. There are several variations on the matrix form of RFA. The RFA formulation, applied in this paper, called *Minimum-State* (MS) RFA formulation, allows to approximate $\hat{\mathbf{Q}}(p)$ with small number of lag terms and maintain high accuracy of approximation.

$$\hat{\mathbf{Q}}(p) = \mathbf{A}_0 + p\mathbf{A}_1 + \mathbf{D}(p\mathbf{I} - \mathbf{R})^{-1}\mathbf{E} \tag{3}$$

Where $\mathbf{R}$ is diagonal matrix of the lag coefficients λ_ℓ. The resulting state-space equation is

$$\begin{bmatrix}\ddot{\mathbf{q}}\\ \dot{\mathbf{q}}\\ \dot{\mathbf{x}}_a\end{bmatrix} = \begin{bmatrix}-\mathbf{M}^{-1}[\mathbf{C}-(B/U)\mathbf{V}_q\mathbf{A1}] & -\mathbf{M}^{-1}[\mathbf{K}-\mathbf{V}_q\mathbf{A0}] & \mathbf{M}^{-1}\mathbf{V}_q\mathbf{D}\\ \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & (U/B)\mathbf{E} & (U/B)\mathbf{R}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}\\ \mathbf{q}\\ \mathbf{x}_a\end{bmatrix} \tag{4}$$

The augmented state vector contains new states known as *aerodynamic states* and they are represented by vector $\mathbf{x}_a$. The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{D}$, $\mathbf{E}$ (Eq. 4) are linearly optimized in a least squares sense while the λ_ℓ terms are optimized by nonlinear nongradient optimizer (Nelder-Mead, Sequential Simplex) with imposed side constrains.

3. ACTIVE CONTROL OF BRIDGE DECK: The control surfaces are attached below the both edges of the bridge deck (Fig.1). The pitch of the control surfaces is actively controlled so as to generate the stabilizing aerodynamic forces. The aerodynamic forces on the control surfaces, calculated through Theodorsen's function, and the experimentally obtained flutter derivatives of the bridge deck (cross section proposed for Akashi Kaikyo Bridge, model geometrical scale 1:150) are

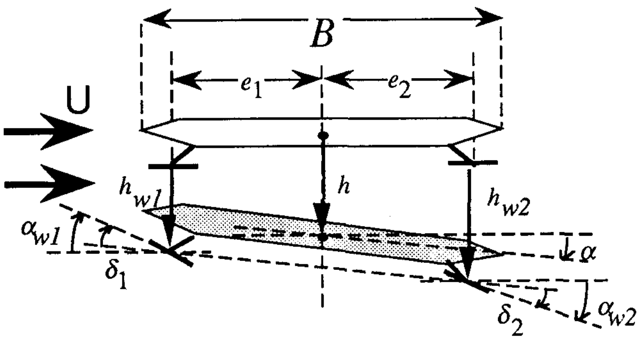

Fig. 1 Bridge Deck with Control Surfaces

both approximated by the MS RFA formulation with two lag terms. The space-state equation of the system is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad (5)$$

where $\mathbf{x} = [\dot{\mathbf{q}}, \mathbf{q}, \mathbf{x}_a]^T$, $\mathbf{x}_a$ is the vector of aerodynamic states of size 6 and $\mathbf{q} = [h/B\ \alpha\ \delta_1\ \delta_2]^T$. The coefficients of matrix **A** are functions of wind speed. The flutter wind speed calculated through complex eigenvalue analysis of the open loop system (no control) is found to be Uf = 10.2 m/s. Considering state feedback control, a control vector is given by $\mathbf{u} = -\mathbf{K}\mathbf{x}$. The gain matrix **K** is found by applying optimal regulator theory as $\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}\mathbf{S}$, where **R** is a weighting matrix and **S** is a solution of steady-state matrix Riccati equation. The gain matrix was calculated for wind velocity of 15 m/s and the root locus of the system with control for wind speed range of 10 to 20 m/s is shown in Fig.2. The arrows in Fig.2 shows the direction of the movement of the eigenvalues with increasing wind speed. The flutter wind speed is increased to Uf = 18.9 m/s. The time response of controlled bridge deck subjected to initial disturbance and wind of speed 15 m/s is shown in Fig.3. The flutter wind speed can be increased further by calculating **K** for large wind speed. A time domain dynamic output feedback control law is now under consideration.

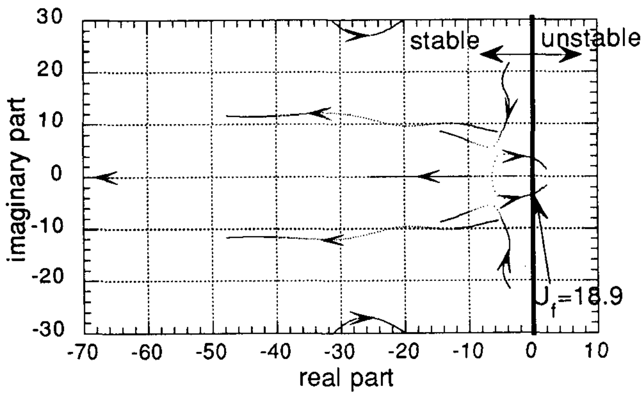

Fig.2 Root Locus of Eigenvalues of Controlled Bridge

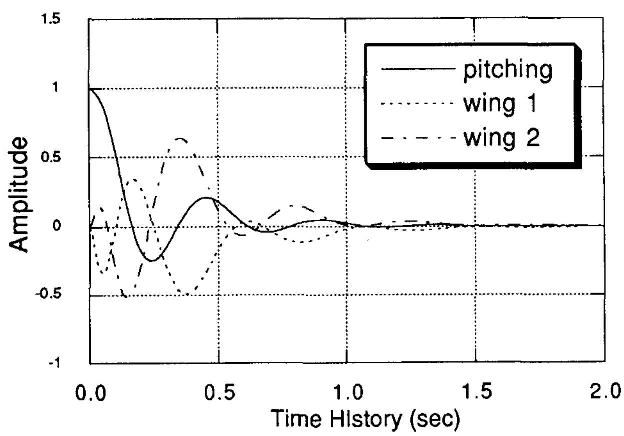

Fig.3 Time History of Controlled Bridge Deck

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