I - 466

ON THE ACCURACY OF STIFFNESS MATRIX MEASUREMENT BY MEANS OF STATIC LOADING TESTS

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1. Introduction. The stiffness matrix represents the fundamental characteristics of a multi-degree-of-freedom structural system. In some types of experiments (e.g. pseudodynamic test), determination of the stiffness matrix of the test structure is often required. For this purpose, direct stiffness matrix measurement procedures via static loading to the test structure, and the accuracy evaluation of the measurement results are presented.

2. Stiffness Measurement Procedures. Let n be the number of degrees of freedom (DOFs) associated with the test structure. When the assumption of a linear-elastic structure is valid, the stiffness matrix $\mathbf{K}$ (dimension $n\times n$) is defined to be the matrix that relates the displacement vector $\mathbf{x}$ (dimension = n) and the restoring force vector $\mathbf{r}$ (dimension = n) by

$$\mathbf{Kx} = \mathbf{r}\,. \tag{1}$$

In order to measure $\mathbf{K}$, a static loading test consisting of n loading steps using different loading condition (imposed displacement or load), is carried out. The restoring forces and displacements are measured at each loading step, as n-dimensional vectors, and the matrix for the restoring force data, $\mathbf{R}$ (dimension=$n\times n$) and the matrix for the displacement data $\mathbf{X}$ (dimension=$n\times n$) are formed by assigning those n-dimensional measurement vectors at n loading steps to the matrices columnwise. From Eq. (1), those matrices should satisfy $\mathbf{KX}=\mathbf{R}$. If the (nonsingular) matrices $\mathbf{X}$ and $\mathbf{R}$ are known, $\mathbf{K}$ can be uniquely determined by

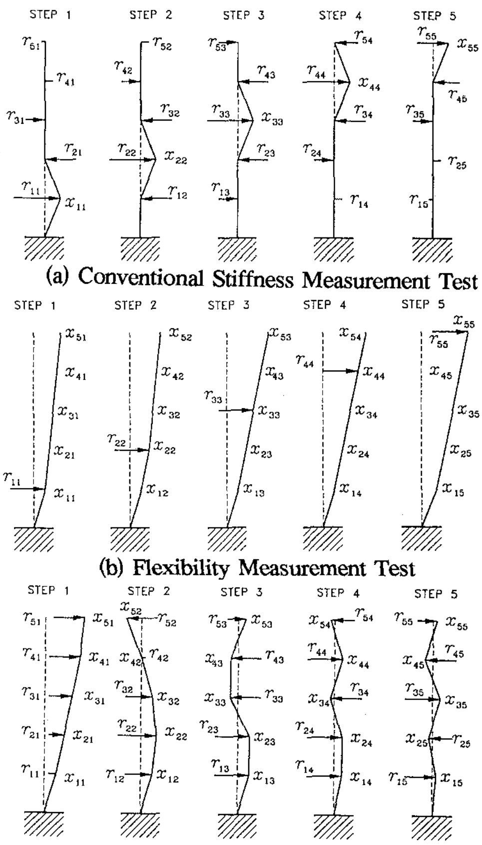
$$\mathbf{K} = \mathbf{RX}^{-1}\,. \tag{2}$$

Typical examples of the combination of displacements, $\mathbf{X}$, or restoring forces, $\mathbf{R}$ to calculate Eq. (2) are as follows: (a) *Conventional Stiffness Measurement Test.* At each loading step, a small displacement is given to only one of the DOFs, while the displacements for the other DOFs are maintained at zero. (b) *Flexibility Measurement Test.* At each loading step, a load is given to only one of the DOFs while maintaining zero loads at the remaining DOFs. (c) *Modal Stiffness Measurement Test.* Displacement patterns proportional to the mode shapes are used in the n loading steps. Knowledge on the mode shapes can be obtained by a preliminary stiffness measurement test or an analytical prediction. The concept of each loading procedure for a cantilever-type structure is schematically shown in Fig. 1.

3. Error Bounds for a Stiff Structure. Due to measurement errors, the measurement of $\mathbf{X}$, denoted by $\hat{\mathbf{X}}$, and the measurement of $\mathbf{R}$, $\hat{\mathbf{R}}$ result in the "measured" stiffness matrix $\hat{\mathbf{K}} = \hat{\mathbf{R}}\hat{\mathbf{X}}^{-1}$ which is generally different from the true stiffness matrix $\mathbf{K}$. For the above three types of tests, an approximate bound on the relative error in the measured stiffness matrix using the assumption of a stiff structure [1] is given by

$$\frac{\|\hat{\mathbf{K}}-\mathbf{K}\|_2}{\|\hat{\mathbf{K}}\|_2} \le \frac{n\lambda_n\delta_x}{\|\hat{\mathbf{R}}\|_2} \tag{3}$$

and an approximate bound on the relative error in the



(a) Conventional Stiffness Measurement Test
(b) Flexibility Measurement Test
(c) Modal Stiffness Measurement Test
Fig.1 Static Loading Patterns

flexibility matrix is

$$\frac{\|\hat{\mathbf{K}}^{-1}-\mathbf{K}^{-1}\|_2}{\|\hat{\mathbf{K}}^{-1}\|_2} \le \frac{n\delta_x}{\|\hat{\mathbf{X}}\|_2} \qquad (4)$$

where $\|\cdot\|_2$ denotes the two-norm of a matrix [2], λ_n is the largest eigenvalue of the stiffness matrix **K**, and δ_x is the bound on the displacement measurement error associated with each DOF such that

$$|\hat{x}_{ij}-x_{ij}| \le \delta_x \quad (i,j=1,2,\ldots,n) \ . \qquad (5)$$

If the same magnitude of the load is used for the three cases, it follows from these evaluations that the modal stiffness measurement test leads to the highest accuracy of the measurement result [1].

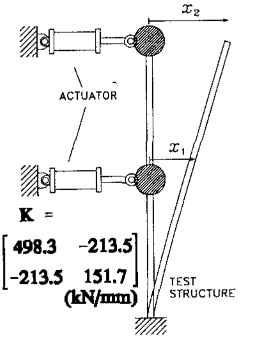


Fig. 2 Two-DOF Structural System Model

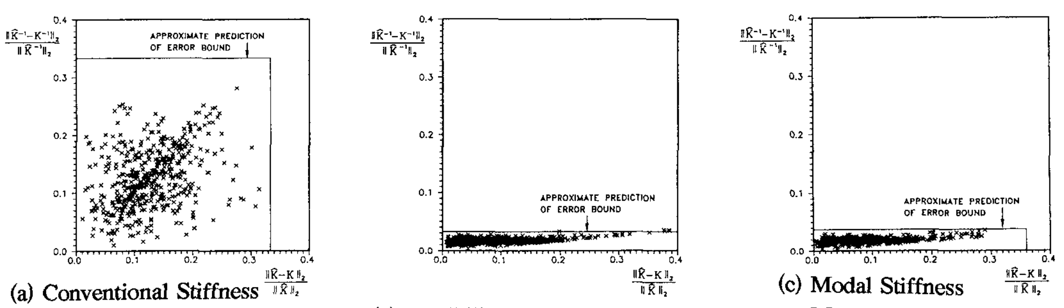
4. Numerical Example. The above three cases are examined numerically using the two-DOF model shown in Fig. 2. The displacement and restoring force patterns used in each type of the test are shown in Table 1. The displacement measurements in $\hat{\mathbf{X}}$ contain artificially generated random measurement errors within ±0.01mm. The values of $\hat{\mathbf{R}}$ shown in Table 1 are computed by $\hat{\mathbf{R}}=\mathbf{K}\hat{\mathbf{X}}$. For each case, 500 stiffness measurement test simulations are performed, and each simulation is represented by a sample point in Fig. 3, representing the relative errors in the stiffness matrix and in the flexibility matrix. The predicted bounds on the relative errors, based on the measured restoring forces and displacements, are also indicated in Fig. 3.

Table 1 Displacements and Restoring Forces for Two-DOF Model Example

case	$\hat{\mathbf{X}}$ (mm)		$\hat{\mathbf{R}}$ (kN)	
(a)	0.0602 0.0	0.0 0.0602	30.00 -12.85	-12.85 9.13
(b)	0.1516 0.2135	0.2135 0.4983	30.00 0.0	0.0 30.00
(c)	0.2857 0.6000	-0.0500 0.0238	14.28 30.00	-30.00 14.28

Note: δ_x=0.01mm, n=2, λ_2=600kN/mm.

Large relative errors in the flexibility matrices shown in Fig. 3a imply that the lower mode information may be lost in the conventional stiffness measurement tests. In the flexibility measurement tests, much lower relative errors in the flexibility matrices, and slightly less relative errors in the stiffness matrices are obtained, see Fig. 3b. This can be translated into the higher accuracy of the higher mode information in the measured stiffness matrix. As shown in Fig. 3c, the result of the modal stiffness measurement tests exhibits an overall tendency similar to Fig. 3b with smaller relative errors in the stiffness matrices.



(a) Conventional Stiffness Measurement Test (b) Flexibility Measurement Test (c) Modal Stiffness Measurement Test

Fig. 3 Simulated Stiffness Matrix Measurement Errors

5. Concluding Remarks. Laboratory testing of a full-scale five-story reinforced masonry building [1] also indicated the advantage of the modal stiffness measurement test. When a reliable measurement of the higher mode information is required, the modal stiffness measurement is the most suitable procedure.

References. [1] A. Igarashi: On-Line Computer Controlled Testing of Stiff Multi-Degree-of-Freedom Structural Systems under Simulated Seismic Loads, *Ph.D Dissertation*, Univ. of Calif., San Diego, 1994. [2] G. H. Golub and C.F. Van Loan: Matrix Computations (2nd Ed.), Johns Hopkins Univ. Press, 1989.