I - 302 Thin Plate Bending of Dissimilar Half Planes With an Elliptical Hole and Debonding on the Interface

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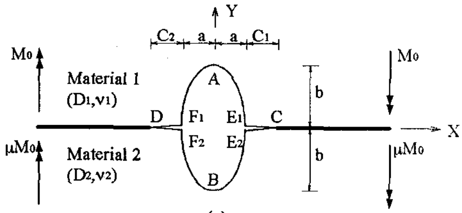
Introduction

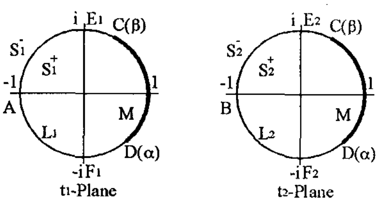
The problem of thin plate bending of two bonded half–planes with an elliptic hole and debonding on the interface is presented. A uniformly distributed bending moment is applied at infinity in the X–direction, parallel to the interface. The complex stress functions approach together with the rational mapping function technique are used in the analysis. Distributions of bending and torsional moments are shown.



(a)

Analytical Method

Fig.1(a) shows two dissimilar half–planes containing an elliptical hole with two interfacial cracks on its both sides, whose lengths are indicated by C_1 and C_2 (See Fig. 1(a)). A mapping function by means of which materials 1 and 2 are mapped into the unit circles of the t_j–planes of Fig.1(b), $j = 1,2$, respectively, is expressed as follows[1]:



(b)

Fig.1(a) Physical Plane, (b) Unit Circles

$$z_j = \omega(t_j) = \frac{E_0}{1-t_j} + \sum_{k=1}^{N} \frac{E_k}{\zeta_k - t_j} + E_c \tag{1}$$

The complex stress functions $\phi_j(t_j)$, $\psi_j(t_j)$, (j=1,2) are regular inside the unit circle of Fig.1(b). Since uniformly distributed bending moment is applied at infinity in the X–direction is considered, the stress functions are expressed as follows:

$$\phi_j(t_j) = \phi_j^A(t_j) + \phi_j^B(t_j), \qquad \psi_j(t_j) = \psi_j^A(t_j) + \psi_j^B(t_j) \tag{2}$$

where $\phi_j^A(t_j)$, $\psi_j^A(t_j)$ represent the stress states at infinity and are given by

$$\phi_1^A(t_1) = \frac{M_0}{4D_1(1+\nu_1)}, \phi_2^A(t_2) = \frac{-\mu M_0}{4D_2(1+\nu_2)}, \psi_1^A(t_1) = \frac{M_0}{2D_1(1-\nu_1)}, \psi_2^A(t_2) = \frac{-\mu M_0}{4D_2(1-\nu_2)} \tag{3}$$

where, $\mu = \dfrac{D_2(1-\nu_2^2)}{D_1(1-\nu_1^2)}$, which accounts for continuity of the rotations at infinity.

The boundary conditions of the external force and displacement are expressed as follows [2]:

$$-\kappa_j \phi_j(\sigma) + \frac{\omega(\sigma)}{\overline{\omega'(\sigma)}} \overline{\phi_j'(\sigma)} + \overline{\psi_j(\sigma)} = \frac{1}{D_j(1-\nu_j)} \left[\int_0^s \left(m(s) + i \int_0^s p(s) ds \right) dz + i a_j z + b_j \right] \tag{4}$$

$$\phi_j(\sigma) + \frac{\omega(\sigma)}{\overline{\omega'(\sigma)}} \overline{\phi_j'(\sigma)} + \overline{\psi_j(\sigma)} = \left(\frac{\partial w_j}{\partial x_j} + \frac{\partial w_j}{\partial y_j} \right) \tag{5}$$

where $\kappa_j = (3+\nu_j)/(1-\nu_j)$, ν_j is Poisson's ratio, D_j is the flexural rigidity. The integral with respect to s represents integration along the boundary line. $m(s)$ and $p(s)$ are the bending moment and bending force per unit length along the boundary line, respectively. a_j and b_j are the real and complex constants of integration, respectively. If a traction free boundary exists, $\psi_j(t_j)$ is given by analytic continuation as follows:

$$\psi_j(t_j) = \kappa_j \overline{\phi_j}(1/t_j) - \frac{\overline{\omega}(1/t_j)}{\omega'(t_j)} \phi_j'(t_j) \tag{6}$$

The boundary condition on L_j is determined by substituting from (6),(3),(2) in (4) and is expressed in terms of the limit values of the function $\phi_j^B(t_j)$ as follows:

$$\phi_j^{B+}(\sigma)-\phi_j^{B-}(\sigma)=\frac{-1}{\kappa_j D_j(1-\nu_j)}\left[\int_{L_j}\left(m(s)+i\int_{L_j}p(s)ds\right)dz\right]\equiv h_{L_j}(\sigma) \quad (7)$$

The boundary conditions on M are the continuation of moments and rotations and is expressed by

$$\phi_1^{B+}(\sigma)+\lambda_1\phi_1^{B-}(\sigma)=\gamma_1 g_1(\sigma)+h_{1M} \quad (8a)$$

$$h_{1M}(\sigma)=\frac{\kappa_2 D_2(1-\nu_2)}{\kappa_2 D_2(1-\nu_2)+\kappa_2\kappa_1 D_1(1-\nu_1)}D_1(\sigma) \quad (8b)$$

$$\lambda_1=\frac{\kappa_1\kappa_2 D_2(1-\nu_2)+\kappa_1 D_1(1-\nu_1)}{\kappa_2 D_2(1-\nu_2)+\kappa_2\kappa_1 D_1(1-\nu_1)} \quad (8c)$$

$$\gamma_1=\frac{(1+\kappa_2)\kappa_1 D_1(1-\nu_1)}{\kappa_2 D_2(1-\nu_2)+\kappa_2\kappa_1 D_1(1-\nu_1)} \quad (8d)$$

The problem of obtaining $\phi_1^B(t_1)$ is reduced to the Riemann–Hilbert problem of (7) on L_j and (8a) on M. Similarly, the function $\phi_2^B(t_2)$ is obtained by merely interchanging the subscripts 1 and 2 in the foregoing derivations.

Derivation of General Solution

The general solution of Riemann–Hilbert problem is first derived for $\phi_1^B(t_1)$ is [3]

$$\phi_1^B(t_1)=H_1(t_1)+\frac{\gamma_1\chi_1(t_1)}{2\pi i}\int_M\frac{g_1(\sigma)}{\chi_1^+(\sigma)(\sigma-t_1)}d\sigma+\chi_1(t_1)P_1(t_1) \quad (9)$$

Since, no loads apply on L_j and M then $H_1(t_1)=0$. The complex stress functions $\phi_1^B(t_1)$ is expressed by

$$\phi_1^B(t_1)=\frac{\gamma_1}{1+\lambda_1}\left[\frac{1}{\kappa_1}\sum_{k=1}^{N}\left\{1+\frac{1+\lambda_1-\gamma_1}{\gamma_1}\frac{\chi_1(t_1)}{\chi_1(\zeta_k)}\right\}\frac{\overline{A_{1k}}B_k-\dfrac{M_0E_k}{2D_1(1-\nu_1)}}{\zeta_k-t_1}\right.$$
$$\left.+\frac{\kappa_2 D_2(1-\nu_2)}{\kappa_1 D_1(1-\nu_1)}\frac{1}{\kappa_2}\sum_{k=1}^{N}\left\{1-\frac{\chi_1(t_1)}{\chi_1(\zeta'_k)}\right\}\frac{A_{2k}\overline{B_k}\zeta_k'^2+\dfrac{\mu M_0\overline{E_k}\zeta_k'^2}{2D_2(1-\nu_2)}}{\zeta'_k-t_1}\right] \quad (10)$$

where $B_k=E_k/\overline{\omega'(\zeta'_k)}$; $\zeta'_k=1/\overline{\zeta_k}$; $A_{jk}=\phi'_j(\zeta'_k)$ $(k=1,2,\ldots,N)$ are obtained by solving 4N simultanous linear equations with respect to the real and imaginary part of A_{jk}.

Conclusions

A closed form solution to the problem of thin plate bending of partially bonded half–planes with an elliptic hole and debondings on its both sides is obtained. Distributions of bending and torsional moments are shown in Fig.2 for a rigidity ratio of $D_2/D_1=0.50$ and Poisson's ratio of material 1 is $\nu_1=0.50$ and that of material 2 is $\nu_2=0.25$ and the debonding lengths are $C_2=C_1=1.0$.

References

[1]Hasebe et. al 1992, ASME, J. of App. Mech.,Vol.114,pp77–83
[2]Savin, 1961, *Stress Concentration Around Holes*, Pergamon Press, Oxford.
[3]Muskhelishvili, 1963, *Some Basic Problems of the Mathematical Theory of Elasticity*, Noordhoff, Groningen.

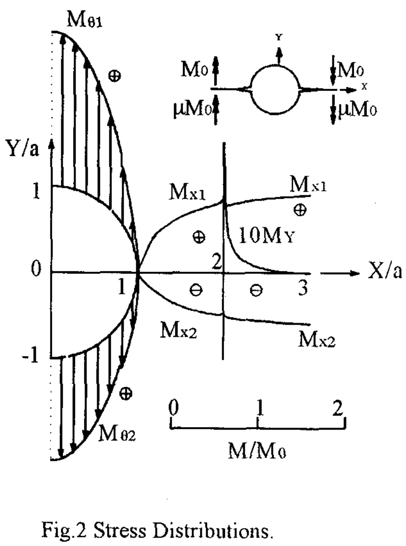


Fig.2 Stress Distributions.