I - 285 AXIALLY SYMMETRIC THERMAL STRESS OF AN EXTERNAL CRACK UNDER GENERAL TEMPERATURE

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Introduction

Kassir and Sih [1] have solved a problem of external circular crack in an isotropic and homogeneous solid under symmetrical temperature conditions. A method is given in [2] to solve an external crack problem under general mechanical loadings (not necessarily self-equilibrating). In this small article a problem of external crack in the infinite solid under general mechanical and temperature conditions has been presented. Since the problem under mechanical loads has been already solved, without loss of generality we assume mechanical loads are zero. Combined effect can be got by superposing the solutions. Equations of elastic equilibrium have been solved using Hankel transforms and Abel operators of the first kind. A closed form solution is obtained. Expressions for stress intensity factors and displacements are presented. In special cases, the results are compared with those available in the literature.

Solution of the Problem

The problem is solved in two steps. In the first step, the stress, displacements and temperature fields are derived in terms of Abel transforms of jumps of stress, displacements, temperature, heat flux at the crack plane $z=0$ given by

$$\int_0^\rho \frac{r[\theta^{(1)}(r,0)-\theta^{(2)}(r,0)]dr}{\sqrt{(r^2-\rho^2)}}=E(\rho),\rho>0 \qquad (1)$$

$$\int_0^\rho \frac{r[\frac{\partial}{\partial z}\theta^{(1)}(r,0)-\frac{\partial}{\partial z}\theta^{(2)}(r,0)]dr}{\sqrt{r^2-\rho^2}}=F(\rho),\rho>0 \qquad (2)$$

together with (3.1)−(3.4) of Ref [3] .

Where superscripts 1 and 2 denote temperature function for $z>0$ and $z<0$ respectively. In the second step the jumps A,B,C,D [3] and E,F are obtained satisfying the axisymmetric boundary conditions of the external circular crack on the plane $z=0$. We suppose that an external circular crack is located in the plane $z=0$ of an infinite, homogeneous and isotropic elastic solid. In terms of the cylindrical polar coordinates (r,ϕ,z), crack may be defined as $r>a,0\leq\phi\leq 2\pi,z=0$. Crack surfaces are subjected to general surface tempertaure and general mechanical loadings. Stress, displacements, temperature and heat flux functions are assumed to be continuous outside the crack $0<r<a(z=0)$. The continuity and the boundary conditions on the plane $z=0$ may be written

$$\theta^{(1)}(r,0)=\theta^{(2)}(r,0);\frac{\partial}{\partial z}\theta^{(1)}(r,0)=\frac{\partial}{\partial z}\theta^{(2)}(r,0),0\leq r<a \qquad (3)$$

$$\theta^{(1)}(r,0)-\theta^{(2)}(r,0)=T_1^*(r);\theta^{(1)}(r,0)+\theta^{(2)}(r,0)=T_2^*(r),r>a \qquad (4)$$

together with equations (17)−(22) of Ref [2] . Since the problem of external crack under general mechanical loads has been solved in Ref [2] and hence temperature problem is considered.

Using the limiting values of stress, displacements and temperature fields as $z\to 0+$ and as $z\to 0-$ and boundary conditions, the problem is reduced to that of solving Abel type of integral equations. Closed form solution is obtained for unknown functions A,B,C,D,E,F . Substituting these values, a simplified expressions for stress, displacement components can be obtained on the crack plane in terms of prescribed functions T_1^*,T_2^* . Using the definitions of Ref [2] , mode-I stress intensity factor K_I can be written

$$K_I=-\frac{\mu(1+\nu)\alpha}{\pi(1-\nu)\sqrt{a}}\int_a^\infty\frac{sT_2^*(s)ds}{\sqrt{s^2-a^2}} \tag{5}$$

and mode-II stress intensity factor is found to be zero.The normal components of the displacement can be written as

$$u_z^{(1)}(r,0)-u_z^{(2)}(r,0)=-2(1+\nu)\frac{\alpha}{\pi}\int_a^r\frac{dt}{\sqrt{r^2-t^2}}\int_t^\infty\frac{sT_2^*(s)ds}{\sqrt{s^2-t^2}},r>a \tag{6}$$

$$u_z^{(1)}(r,0)+u_z^{(2)}(r,0)=-2(1+\nu)\frac{\alpha}{\pi}\int_{max(r,a)}^\infty\frac{dt}{\sqrt{t^2-r^2}}\int_a^t\frac{sT_1^*(s)ds}{\sqrt{t^2-s^2}},r>0 \tag{7}$$

where α is coefficient of linear expansion of the solid, ν and μ are poisson's ratio and modulus of shear rigidity respectively. For a special case in which crack surfaces are subjected to constant temperature over a circular ring of inner and outer radii a and c respectively, that is ,

$$T_1^*(r)=H(c-r)\tau_1;T_2^*(r)=H(c-r)\tau_2,r>a \tag{8}$$

where $H(r)$ is Heaviside step function.The definition (8) indicates that the temperature applied on the upper face $(r>a,z=0+)$ is different from the lower face $(r>a,z=0-)$ of the crack. In this case stress intensity factor K_I becomes

$$K_I=-\frac{\mu(1+\nu)\alpha\tau_2\sqrt{c^2-a^2}}{\pi(1-\nu)\sqrt{a}} \tag{9}$$

Similarly displacement components also can be simplified and they are found to be in terms of the elliptic integrals of the first and the second kind.

Conclusions

(i) A special case in which the temperature applied on the faces of the crack is same (symmetrical about the plane $z=0$), results are compared with those available in the literature. (ii) Vanishing of mode-II stress intensity factor K_{II} may be attributed to the axisymmetry. (iii)In similar lines, the problem of external crack under general heat flux conditions can be solved, but flux problem may be more comlex than temperature problem.

References

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