III - 390 Gravitational Stresses in Anisotropic Elastic Medium

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1. Introduction

In this paper, there is an attempt to present closed form solutions for the stresses induced by body forces due to gravity.

A similar study was conducted among others by Amadei et al. [1,2] who presented solutions for the stress field induced by gravitational loading of laterally restrained anisotropic rock mass with a horizontal ground surface. These solutions were limited to orthotropic and transversely isotropic rock mass with horizontal or vertical planes of symmetry. Amadei and Pan [3] considered the gravity-induced stress field in a homogeneous orthotropic and transversely isotropic rock mass with strata which are inclined with respect to a horizontal ground surface.

The purpose of this paper is to study the stresses induced by gravity forces in homogeneous, orthotropic and transversely isotropic, rock mass with strata arbitratily inclined with respect to the boundary, including out-of-plane inclination. The surface of the rock mass itself is now taken inclined with respect to the horizontal.

2. Analytical Solution

2.1 Constitutive equation

The constitutive model for the rock mass is assumed to be described by Hooke's law. The elastic constants, which enter into equations of the generalized Hooke's law of an anistropic body, depend on the direction of the coordinate axes. Only in the case of an isotropic body are the constants invariant in any orthogonal coordinate system.

Let an (x', y', z') coordinate system be attached to the principal axes of elastic symmetry and let's consider a new coordinate system (x, y, z) attached to the overall structure (global coordinate system). A more general transition from the (x', y', z') coordinate sytem to the (x, y, z) system is performed using three angles (Figure 1). These are defined as follows:

(a) ϕ: Rotation about the x'-axis → $(\hat{x}, \hat{y}, \hat{z})$ coordinate system,
(b) β: Rotation about the $\hat{y}$-axis → $(\tilde{x}, \tilde{y}, \tilde{z})$ coordinate system,
(c) α: Rotation about the $\tilde{z}$-axis → (x, y, z) coordinate system.

Substituting the direction cosines defining the position of the new coordinate system (x, y, z) with respect to (x', y', z') into the transformation formulas [4], the new elastic constants can be obtained.

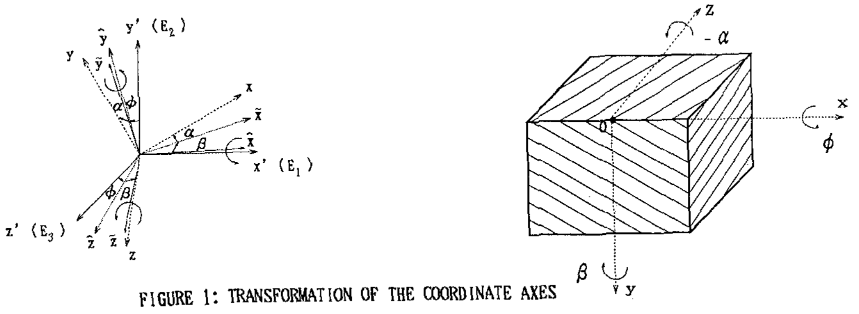


FIGURE 1: TRANSFORMATION OF THE COORDINATE AXES

2-b Formulation of the stresses

We consider the equilibrium of a semi-infinite half space, of uniform density ρ, representing a rock mass under gravity. The surface of the rock mass is assumed to be inclined by an angle ζ with respect to the horizontal (Figure 2), hence the body force has two components along the x and y-axes: $f_x = \rho\ g\ \sin(\zeta)$ and $f_y = \rho\ g\ \cos(\zeta)$ respectively, where g is the acceleration of gravity.

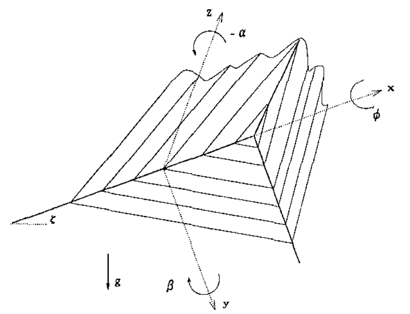


FIGURE 2: GRAVITATIONAL STRESSES IN ANISOTROPIC MEDIUM.

Since the body forces here are of gravitational nature, all the components of stress, strain and displacements can be assumed to be independent of x and z directions. Accordingly, there is no lateral strain and all ε_x, ε_z and γ_{xz} vanish.

Manipulating the constitutive equations together with the equilibrium equations and bearing in mind the condition of no lateral strain, the components of stress can be computed.

Under a condition of no lateral strain, the magnitude of the vertical stress is independent of the rock mass elastic properties and depend only on the depth. The two horizontal principal stresses, however, depend largely on the type and degree of the rock mass anisotropy and, unlike the results obtained by the classical isotropic solution of Terzaghi and Richart [5], they are not always equal.

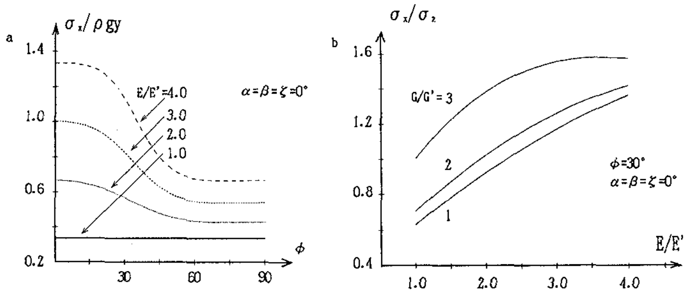
3. Parametric Study

A parametric study is carried out to show the effect of the elastic properties of the anisotropic rock mass and the type and degree of anisotropic orientation on the stresses induced by gravity. Figure 3 shows the effect of the out -of-plane inclination (ϕ), the young modulus ($E=E_x=E_z$, $E'=E_y$) and the shearing modulus ($G=G_x=G_z$, $G'=G_y$) on the variation of stresses. The following results are obtained:
a) For the general case of anisotropic rock mass, the horizontal normal stresses are not always equal.
b) The horizontal normal stresses depend largely on the elastic constants of the rock mass, the orientation of anisotropy and the inclination of the boundary.

FIGURE 3

a) VARIATION OF $\sigma_x/\rho gy$ WITH ϕ,

b) VARIATION OF σ_x/σ_z WITH E/E' AND G/G'.



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