II-454 Mathematical Model for the Prediction of Dissolved Oxygen in Lakes

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1. Introduction

The accurate prediction of dissolved oxygen(DO) in lakes specially in eutrophic lakes is of paramount importance with regards to the water quality. The temporal variation of DO in the lake Calhoun is studied with the aid of a numerical model. The DO level can be used to estimate the quality of the water.If the water quality is highly degraded, the DO depletion may cause fish kills and anoxic conditions for other living organisms in the ecosystem. The main factors that affect DO budget are water temperature and the amount of organic matters present. The rate of change in DO concentration is primarily dependent on the consumption and production parameters. This model formulation comprises of carbonaceous and nitrogenous oxidation, reaeration with atmosphere, photosynthesis, algal respiration, sediment oxygen demand, zooplankton respiration and fish respiration.

2. Model Formulation

$$\frac{dC}{dt} = -k_1L - 4.57k_N N + k_2(C_s - C) + a_p\mu P - a_p rP - SOD - k_F\left[\frac{F}{H}\right] - a_z k_z Z \quad (1)$$

where C is DO concentration; P is phytoplankton concentration; N is ammonia concentration; L is first order ultimate carbonaceous oxygen demand; C_S is saturated DO concentration; SOD is sediment oxygen demand; k_1 is biochemical deoxygenation rate; k_N is nitrification rate; k_2 is reaeration rate; a_p is the ratio of oxygen production to uptake per algal mass; m is phytoplankton growth rate; r is phytoplankton respiration rate; k_F is fish respiration rate; H is mean depth of the lake; a_Z is the ratio of oxygen production to uptake per unit mass of zooplankton carbon; k_Z is half saturation constant for zooplankton filtering efficiency and Z is zooplankton concentration.

3. Carbonaceous and Nitrogenous BOD

$$L = (BOD_u)e^{-k_1 t} \quad (2)$$

$$k_1(T) = k_{20}\theta^{T-20} \quad \text{and} \quad k_N(T) = k_{N20}\theta^{T-20} \quad (3.a.b)$$

where T is the temperature in centigrade.

4. Reaeration

The following equations are used to find aeration coefficient which was earlier used by Thomann & Mueller and saturation oxygen concentration.

$$k_2(20) = \frac{0.728W^{0.5} - 0.317W + 0.0372W^2}{H} \quad (4)$$

$$k_2(T) = k_{20}\theta^{T-20} \quad (5)$$

$$C_s = 1.43[(10.291 - 0.2809T + 0.006009T^2 - 0.0000632T^3)] \quad (6)$$

where W is wind speed in m/s and H is mean depth in meters.

5. Photosynthesis, Algal Respiration and SOD

The phytoplankton growth rate,respiration rate and SOD are determined from the equations below.

$$\mu = \mu_{max} \exp(aT + b) \frac{DL}{24} \frac{N}{N + N_k} \frac{I}{I_s} \exp(1 - \frac{I}{I_s}) \tag{7}$$

$$I = I_0 \exp(-kZ) \quad \text{and} \quad k = k_w + k_c P \tag{8.a.b}$$

$$r = k_r \theta^{t-20} \tag{9}$$

$$SOD = \mu_\beta \frac{C}{k_{o_2} + C} + k_c C \tag{10}$$

where k_s is maximum growth rate; a and b are constants; DL is the daily solar hours for a given day; N is the nutrient concentration; N_k is half saturation constant; I_s is the solar radiation intensity at surface (Photosynthetically Active Radiation-PAR); k_w is light attenuation coefficient due to water color and to dissolved and nonplanktonic suspended solids; k_c is light attenuation coefficient due to phytoplankton; I_o is the optimum light intensity; and μ_β, k_c and k_{o_2} are respiration rate constants.

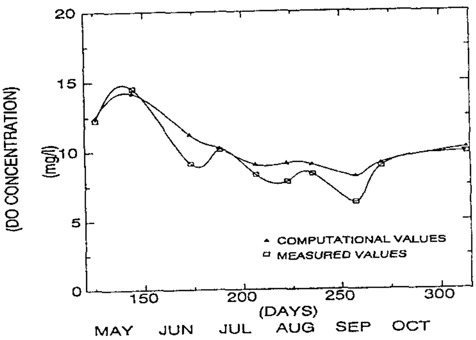

Fig. 1.

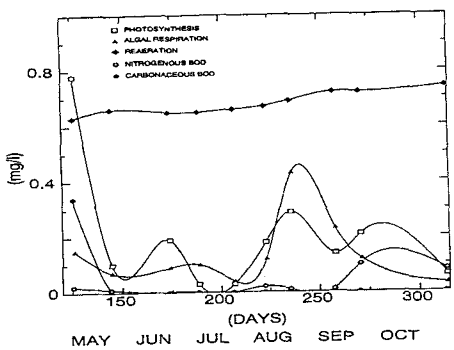

Fig. 2.

6. Results and Discussions

The Crank Nicolson Finite Difference Scheme was used to calculate the temporal variation of DO. Fig.1 shows the comparison of computational and measured values of DO at the surface. Fig.2 shows each component separately. The model was built on a well established concepts as well as in situ field measurements.Model simulations agree favorably with observations. The fig.2 manifests that the reaeration is predominant at the surface where as photosynthesis and algal respiration contribute fairly well.

7. References

1. J.H.W.Lee, R.S.S.Wu, (1991). Dissolved oxygen variation in marine culture zone. *J. Env. Eng.*, Vol.117, No 6.
2. Robert R. Walker, (1986). Model for sediment oxygen demand in lakes. *J. Env. Eng.*, Vol.112, No 1.