II - 449

A NUMERICAL MODEL FOR MUD MASS TRANSPORT UNDER WAVES

Daoxian SHEN, University of Tokyo, Student Member
Masahiko ISOBE, University of Tokyo, Member
Akira WATANABE, University of Tokyo, Member

1. Introduction

Mud mass transport plays an important role on the geological processes in coasts and river mouths. The purpose of the present study is to investigate mud mass transport under waves. Based on an empirical rheology equation, a numerical model is developed to predict the wave attenuation as well as the bed mud motion. Then, numerical simulations for the motions of one- and two-layered mud beds are carried out and the results are compared with laboratory data.

2. Numerical Model of Mud Motion

The interaction between waves and muddy bottom is very complicated mainly because of the complex characteristics of mud, especially, of its rheological relations. Therefore, an empirical rheology equation (Suzuki, 1992) which was proposed on the basis of the experimental results on the rheology (Shen, 1992) is used to describe the relations:

$$\tau = G\varepsilon - \tau_G \tanh(\alpha_G \varepsilon) + \mu\gamma + \tau_0 \tanh(\alpha_\mu \gamma) \quad (1)$$

where τ is the shear stress, ε the shear strain and γ the shear rate. The other parameters are constants of which G and μ represent the elasticity and viscosity, respectively, and $\tau_G \tanh(\alpha_G \varepsilon)$ and $\tau_0 \tanh(\alpha_\mu \gamma)$ represent correction terms.

The linearized equations of motion and the continuity equation for the incompressible mud layer under oscillatory loading are

$$\rho_{mj}\frac{\partial u_j}{\partial t} = -\frac{\partial p_j}{\partial x} + \frac{\partial \tau_j}{\partial y} \quad (2)$$

$$\rho_{mj}\frac{\partial w_j}{\partial t} = -\frac{\partial p_j}{\partial y} + \frac{\partial \tau_j}{\partial x} \quad (3)$$

$$\frac{\partial u_j}{\partial x} + \frac{\partial w_j}{\partial y} = 0 \quad (4)$$

where u_j and w_j are the horizontal and vertical components of the velocity of the $j-$th layer, p_j the pressure, τ_j the shear stress which is represented by the rheological model proposed. The above governing equations can be integrated numerically with the following boundary conditions: i) Non-slip condition at the fixed bottom ($u_n = 0$ and $w_n = 0$); ii) Zero shear stress at the mud surface ($\tau_0 = 0$).

The SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm, which is developed by Patankar (1980), was used for the calculation of the mud motion. Once the oscillatory motion of the mud is solved, the energy dissipation rate, D_w, is calculated by

$$D_w = \overline{\int_0^d \frac{d}{dt}(\tau\varepsilon)\,dz} \quad (5)$$

where d is the thickness of the mud bed and the symbol $\overline{}$ denotes the average over a wave period. The distribution of the wave height in the wave propagation direction can be estimated from D_w. The mass transport velocity consists of the Eulerian average velocity, U_E, and the Stokes drift, U_s. These are generally calculated by

$$U_{sj} = \overline{\frac{\partial u_j}{\partial x}\int u_j\,dt} + \overline{\frac{\partial u_j}{\partial z}\int w_j\,dt} \quad (6)$$

$$\mu^{cj}_{ave}\frac{\partial^2 U_{Ej}}{\partial z^2} = \rho_j\left(\frac{\partial \overline{u_j^2}}{\partial x} + \frac{\partial \overline{u_j w_j}}{\partial z}\right) \quad (7)$$

where μ^{cj}_{ave} is computed only in the layer where μ changes, and defined as

$$\mu^{cj}_{ave} = \mu_{cj} + \frac{1}{\pi}\int_\sigma^{\sigma+\pi} \frac{\tau_{cj}\alpha_\mu}{\cosh^2(\alpha_\mu \gamma_{cj})}\,dt \quad (8)$$

where σ is the phase value where the sign of the shear stress changes.

3. Comparison with Experimental Results

To verify the numerical model, the previous laboratory data are used to compare with the calculated results by the present model. Figure 1 compares the calculated excursion amplitude of the mud particle with the experimental results. A good agreement is seen between the calculation and laboratory data.

The natural deposit processes frequently give rise to layered soil deposits having alternating layers with different properties. In order to predict the mud motion and the processes, the behavior of two-layered mud is studied.

Laboratory experiments were performed in a wave flume of 21.00m long and 0.80m wide. A two-layered muddy bottom was constructed with the lower layer of the consolidated mud of 8.5cm thick and the upper layer of the newly mixed mud of 7.9cm thick. The two-layered bottom is very similar to the bottom deposited for an order of a year in natural coastal areas. The mud mass transport velocity and wave damping were measured.

Fig. 2 compares the calculated and measured mass transport velocity at the beginning of the experiment. The profiles of the transport velocity is similar to each other, but the calculated mud mass transport velocity in the upper layer is larger than the measurement.

Fig. 3 compares the calculated and measured wave height change. The difference may be because the values of parameters are not estimated appropriately.

4. Conclusions

Based on the empirical constitutive model proposed, a numerical model is developed to predict the mud mass transport velocity and wave height change under wave action. Agreement is good for the mass transport velocity of two-layered mud and fairy good for the wave height change.

5. References

[1] Shen, D.-X, M. Isobe and A.Watanabe(1992): Experimental study on the rheology properties of mud, Proc. 47th Annual Conf. of JSCE. Vol. 2, pp. 1208-1209 (in Japanese).

[2] Suzuki, T., M. Isobe and A.Watanabe(1992): Numerical Calculation on mud motion by an empirical rheology model, Proc. 47th Annual Conf. of JSCE. Vol. 2, pp. 1210-1211 (in Japanese).

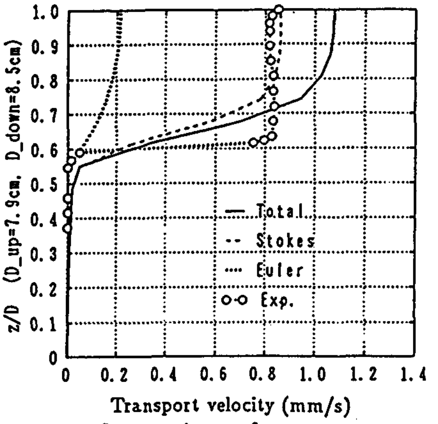

Fig.2 Comparison of mass transport velocity at 2.0m (T=1.02s, h=30cm).

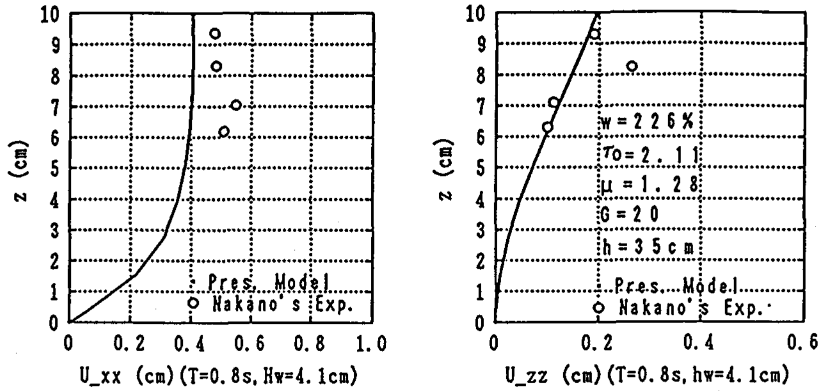

Fig 1 Comparison of excursion amplitude of mud particle.

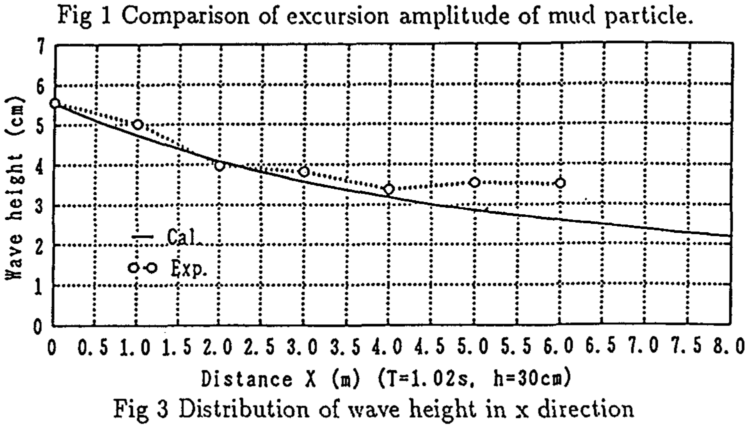

Fig 3 Distribution of wave height in x direction