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Velocity profiles in unsteady flow expressed in power low

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INTRODUCTION

Velocity profiles in boundary layer flow, pipe flow and in open-channel flow may be expressed by the logarithmic law or a power low. While log-law is considered as semi-theoretic, power law is said to be wholly empirical, although it has been widely used (e.x., Toffaleti [3], Karim and Kennedy [1]). Compared with power low which has an unknown exponent, log-law involves several "universal constants" that are not universal (Tu [4], Ch.2), particularly in unsteady flow where they are shown to vary with the longitudinal pressure gradient (Tu [4], Ch.5). Needless to say, it is interesting to examine velocity profiles, particularly those from unsteady flow, using the power law.

In this technical note, existing theory (Eq.3) is found to overestimate the m-values in unsteady flow (Fig.1). Subsequently a theoretical expression for the exponent is derived (Eq.8), which predicts very well the m-values for the velocity profiles measured in both steady and unsteady flows (Fig.2).

THEORETICAL CONSIDERATION AND DATA ANALYSIS

Power law and log-law in their familiar forms are written in Eqs.1 and 2 respectively. From Eqs.1 and 2 an expression (Eq.3) for m is derived by Zimmermann and Kennedy [5]). Here, u is the streamwise velocity component at height y; u_{max}, the maximum velocity at the free surface; D, the water depth; m, power law's exponent; u_*, friction velocity; κ=0.41 is the Karman constant and f, friction coefficient.

$$u = u_{max}\left(\frac{y}{D}\right)^{1/m} = V\left(\frac{m+1}{m}\right)\left(\frac{y}{D}\right)^{1/m} \quad (1)$$

$$\frac{u - u_{max}}{u_*} = \frac{1}{\kappa}\ln\frac{y}{D} \quad (2)$$

$$m = \kappa\sqrt{\frac{8}{f}} \quad (3)$$

$$\frac{\partial u}{\partial y} = \frac{u_{max}}{m}\frac{D}{y}\left(\frac{y}{D}\right)^{1/m} \quad (4)$$

$$\frac{u - u_{max}}{u_*} = \frac{1}{\kappa}\ln\frac{y}{D} - \frac{2\Pi}{\kappa}\cos^2\left(\frac{\pi}{2}\frac{y}{D}\right) \quad (5)$$

$$V = \frac{m}{m+1}u_{max} \quad (6)$$

$$\frac{V - u_{max}}{u_*} = -\frac{1}{\kappa} - \frac{\Pi}{\kappa} \quad (7)$$

$$m = \frac{\kappa}{1+\Pi}\sqrt{\frac{8}{f}} \quad (8)$$

MacQuivey [2] carried out a series of flow measurements with high quality. Tu [4], studying both steady and unsteady flow in a gravel-bed flume, also measured a large number of velocity profiles. These profiles were shown to follow power law, and their m-values were calculated by least square regression (due to space limit the details are not shown here, only the data range are given in Table 1). In Fig.1 the m-values obtained from the velocity profiles are compared with those calculated from Eq.3. It is seen that:

1) the m-values obtained from the profiles are in general smaller than those in alluvial flows, where they are around 6 (see for example, Toffaleti [3]);

2) Eq.3 overestimates the m-values for the velocity profiles measured in unsteady flow.

Equation 4 indicates that the smaller the m-value, the larger would be the velocity gradient. Hence, the first observation means that velocity gradients for the velocity profiles measured in flow over large roughness, particularly near the bottom, are larger than those in flows over smooth bed.

Since Eq.3 overestimates the m-values for unsteady flow, it needs to be improved. This can be done by using power law (Eq.1) and Coles law (Eq.5), as explained in the following.

Coles, examining data obtained in boundary layer flows, found that Eq.5, instead of Eq.2, should be used to describe the velocity profiles. In Eq.5 Π is the so-called wake parameter, which varies with the longitudinal pressure gradient and is about 0.55 in zero pressure gradient boundary layer flows.

Integrating both Eqs.2 and 5 from the bottom to the water surface, and division by D, we obtain easily Eqs.6 and 7. Finally we derive from Eqs.6 and 7 a theoretical formula, Eq.8, for calculating the m-value in Eq.1. With stronger wake one has from Eq.8 smaller m, thus larger velocity gradient. Note also that Eq.8 reduces to Eq.3 if one assumes that no wake exists in the velocity profile under consideration.

Table 1 Data range of the velocity profiles analyzed

Data by	flow	bed	d_s (cm)	width (cm)	depth (cm)	u_* (cm/s)	V (cm/s)	$\sqrt{8/f}$	Π	m
MacQuivey	steady	block	0.32-2.86	61	12.6-12.7	3.0-8.1	30-54	4.8-10.2	0.0-0.1	2.3-4.3
MacQuivey	steady	riverbed	3.81-15.24	122	14.3-28.4	2.1-3.0	20-26	7.8-10.5	0.0-0.45	2.9-3.4
Tu	steady	gravel	1.35-2.3	60	9.1-17.0	6.7-11.9	45-116	6.9-9.7	0.0-0.25	2.4-3.2
Tu	unsteady	gravel	1.35	60	12.2-21.4	3.8-8.4	54-96	8.3-15.7	0.0-0.72	2.6-4.2

Using Eq.8, the data from Fig.1 are replotted in Fig.2, which shows reasonable agreement between the actual m-values and predicted ones. The Π-values in Eq.8 were previously obtained from the velocity profiles. In practice the velocity profile is unknown, so we need to determine in the first place Π-value from known hydraulic parameters. This problem can be solved using an empirical formula [4], though it can not be discussed here due to space limit.

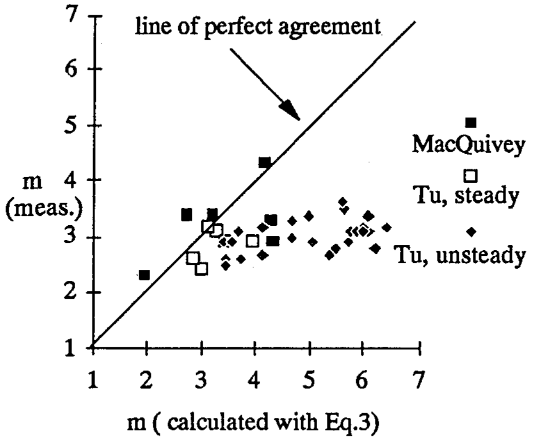

Fig.1 m-values obtained from the velocity profiles and the ones calculated with Eq.3

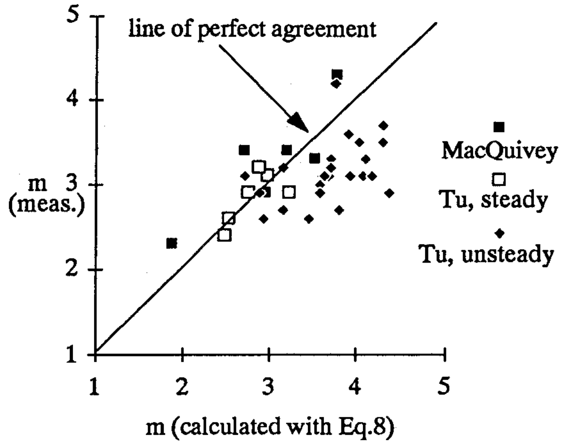

Fig.2 m-values obtained from the velocity profiles and the ones calculated with Eq.8

CONCLUSIONS

Velocity profiles measured in both steady and unsteady flows follow power law (Eq.1). The m-values obtained from the velocity profiles measured in steady flows over large roughness are in reasonable agreement with the ones predicted by Eq.3, and they are in general smaller than the m-values from plain river flows, thus showing larger velocity gradient. For the velocity profiles measured in unsteady flows, the m-values predicted by Eq.3 deviate from the measured ones (Fig.1). Using Coles law (Eq.5), a new relationship is derived for the m-values (Eq.8), which gives very good results (Fig.2).

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