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APPLICATION OF THE LINEAR PERTURBATION MODEL(LPM) FOR FORECASTING INFLOW TO THE SIRIKIT RESERVOIR

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1. INTRODUCTION

Forecasting of inflow to the reservoir is very useful information for reservoir operation especially during the flood. Short term forcast can be done by many techniques depending on the particular problem. Generally, lead time of forecast, say, 1 or 2 days or longer is required for practical purpose. To achieve that the forecast rainfall is also needed for rainfall-runoff model which is, of course, not an easy task. To avoid such complexities of rainfall forecasting, one may use flood routing technique if upstream input(s) can provide enough time to reach the point of interest downstream. This study applies the linear perturbation model(LPM) for multiple input-single output for flood routing to forecast the inflow to the Sirikit reservoir. Both nonparametric (unconstrained) and parametric(transfer function) form of the model are presented.

2.THE LINEAR PERTURBATION MODEL(LPM)

The LPM was originally developed by Nash & Barsi(1983) as a single input-single output rainfall-runoff model. Liang(1986) extended the LPM for multiple input-single output flood routing. The LPM assumes that in any year in which the input exactly follows the seasonal expectation, the output would similarly follows its seasonal expectation, and in other years the departures from the seasonal expectations occurring in the two series would be linearly related. The structure of the LPM is shown in Figure 1. where :

I_T : daily input
I_S : seasonal daily mean input
I_D : daily input departures
Q_T: daily output
Q_S : seasonal daily mean output
Q_D: daily output departures

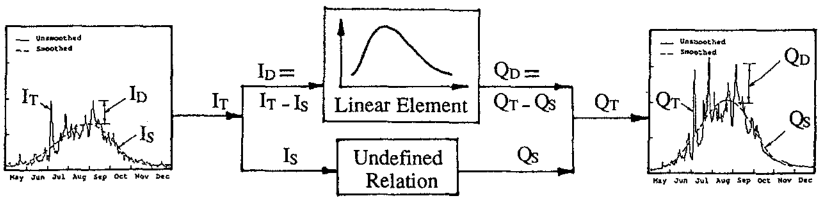

Figure 1 Schematic Representation of The Structure of The LPM

The seasonal daily mean for input and output series,I_S and Q_S are obtained by the seasonal model which uses the mean daily values smoothed by the discrete Fourier series analysis(Salas et al.,1980) throughout the period of calibration. The daily output departures Q_D are calculated by a linear element(shown in the upper box in Figure 1) based on the daily input departures I_D. This study applies two forms of the linear element for the multiple input-single output system. The first one is the nonparametric (unconstrained) form described as OLS/UC which can be expressed in discrete form as follows:

$$y_t = \sum_{k=1}^{L}\sum_{j=1}^{m(k)} x^{(k)}_{t-j+1}\, h^{(k)}_j + e_t \qquad (1)$$

where x_t are the departure inflow series; y_t are the departure outflow series; k is the number of inputs; j is the number of the pulse responses; $h^{(k)}_j$ and m(k) are the ordinates and memory length of the pulse responses for input k; and e_t is the error term. The second one is parametric(constrained) form defined by the transfer function model(LTF).

$$y_t=\sum_{j=1}^{r} \delta_j y_{t-j}+\sum_{k=1}^{L}\sum_{j=1}^{s(k)} \omega^{(k)}_j x^{(k)}_{t-b(k)-j+1}+e_t \qquad (2)$$

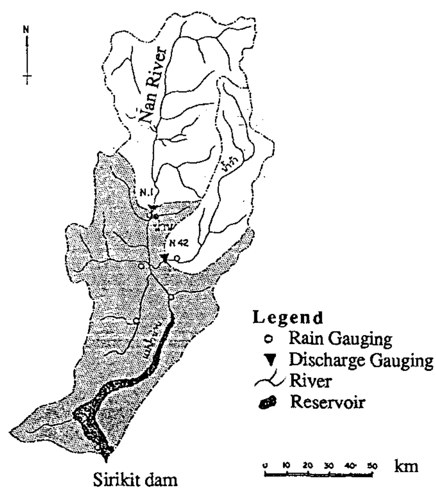

Figure 2 Location Map of The Nan River

where δ_r are the autoregressive parameters; $\omega^{(k)}_s$ are the moving average parameters associated with each input k; b(k) is the pure time delay for each input restricted to integer values only. Both forms of eq. 1 and eq. 2 can be written in matrix notation and then solved by ordinary least squares method. The pulse response for the parametric form can be derived directly from the transfer function parameters(Box & Jenkins,1976).

3. APPLICATION

The LPM is applied to forecast the inflow to the Sirikit reservoir(13,130 sq.km) located in the Nan river in the northern part of Thailand. Figure 2 shows the location map of the Sirikit reservoir. Two upstream discharge gauging stations at N1 and N42 are used as inputs and then routed to the Sirikit reservoir using the LPM. These two stations

N1 and N42 covering the area of 4,609 and 2,107 sq.km are located 155 and 120 km upstream of the forecasting point respectively. The daily flows from 1978 to 1983 are used for calibration while verification are made by the data from 1984 to 1986. Both nonparametric(eq. 1) and parametric(eq. 2) models are applied.

4. THE EFFICIENCY CRITERIA

Nash & Sutcliffe(1970) defined the model efficiency R^2 which is analogous to the coefficient of determination in linear regression as follows:

$$R^2=\frac{(F_0-F)}{F_0}; \qquad F_0=\Sigma(y-\bar{y})^2; \qquad F=\Sigma(y-\hat{y})^2$$

where R^2 is the model efficiency; F_0 is the initial variance; F is the index of residual error; y is the observed discharge; $\bar{y}$ is the mean of observed discharge for the calibration period; and $\hat{y}$ is the estimated discharge.

5. RESULTS AND DISCUSSION

Comparison of pulse responses obtained by both forms for each input N1 and N42 are shown in Figure 3 and 4 respectively. Generally speaking, the memory length(m) for nonparametric form(OLS/UC) is obtained by trial and error choosing the longer m which the last ordinate is still above its standard error. However, this study chooses 5 days as a fixed value for the purpose of investigation. It can be seen that some negative ordinates are found for the last few days and can be ignored actually. For the parametric(LTF) form the adopted order of autoregressive(r) and moving average(s) terms are 2 and 3 respectively with the pure time delay of 0. The pulse responses obtained from both forms are quite similar. Figures 5 and 6 show the comparison of estimated and observed discharge hydrographs at the Sirikit reservoir for OLS/UC and LTF,respectively. The satisfactory agreement are found in both cases. The model efficiency R^2 obtained by OLS/UC are 93.01% and 90.59% for calibration and verification period while those obtained by LTF are 92.81% and 90.24% ,respectively.

6. CONCLUSIONS

The pulse responses for both forms indicate significant ordinate in the following day which is very useful for forecasting in advance. The R^2 obtained from both forms are considered acceptable. However, in real-time operation, the estimated discharge in autoregressive terms of LTF may be replaced with the observed discharge from the past which will improve the accuracy of forecast to some extent. This procedure is called updating operation.

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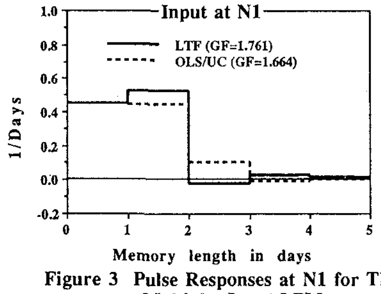

Figure 3 Pulse Responses at N1 for The Multiple Input LPM

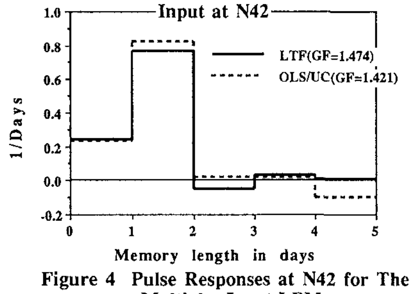

Figure 4 Pulse Responses at N42 for The Multiple Input LPM

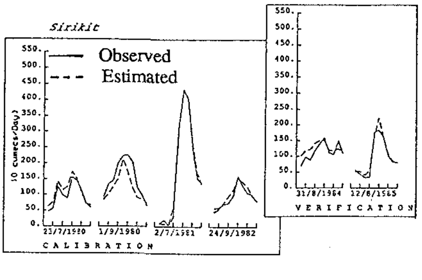

Figure 5 Observed and Estimated Discharge Hydrograph By OLS/UC

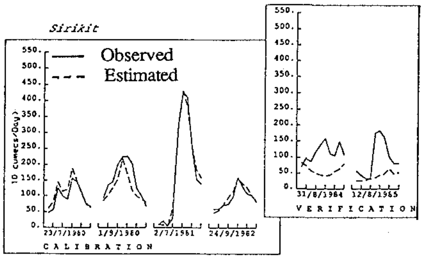

Figure 6 Observed and Estimated Discharge Hydrograph By LTF