I－578 Frequency Domain Criterion of Optimality in Active Control of Structures

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1. Introduction: In active control of structures, feedback gains are generally determined either by linear optimal control or by pole placement methods[1]. These methods can be explained as follows: consider a time invariant n dof system with system matrix **A**, actuator location matrix **B** and control force vector **u**(t):

$$\dot{\mathbf{y}}(t)=\mathbf{A}\mathbf{y}(t)+\mathbf{B}\mathbf{u}(t) \quad (1)$$

In pole placement method, we wish to find a control law u(t)= -**G**y(t), **G** is the feedback gain matrix, by moving the poles of the system in Eq.(1) to the desired locations. On the other hand, in linear optimal control we derive **G** matrix by minimizing a quadratic performance index with state weighting matrix **Q** and control weighting matrix **R**. The relationship between these two methods was first studied by Kalman[2] and is generally known as " the inverse optimal control problem". The gain matrix by pole placement although places the poles at desired locations, it doesn't possess the properties of optimality (e.g., less sensitive to parameter variations etc.). The purpose of inverse optimal control problem is to calculate optimal **G** by deriving **Q** matrix for desired pole locations. Since the gain matrix is unique for a particular location of poles of a single actuator system, the optimal gain matrix can be directly derived by placing the poles in the region of optimal locations obtained by the frequency domain criterion of optimality, instead of trial-and-error selection of **Q**,**R** matrices. However, for a multi-actuator system the gain matrix is non-unique[1]. Hence, to obtain an optimal gain matrix for desired pole locations, we will have to minimize a quadratic performance index using **Q** and **R** matrices obtained by the inverse optimal control.

In this paper, we will present the interpretation of the optimality in frequency domain[3,4] in terms of optimal pole locations. Discussion about calculation of **Q** matrix can be found in ref.[5,6].

2. Frequency Domain Criterion of Optimality:

(A) Single-Actuator Systems: Consider the block diagram of a single actuator system shown in Fig.1. The polynomials of open and closed loop systems and adjoint matrix for open-loop system can be written as,

$$P(s)=\det(s\mathbf{I}-\mathbf{A}),\ P_k(s)=\det(s\mathbf{I}-\mathbf{A}+\mathbf{bg}),\ \Phi(s)=(s\mathbf{I}-\mathbf{A})^{-1} \quad (2)$$

Here, since **B** and **G** matrices are n vectors, we denote them by **b** and **g**. We note from Fig.1 that the return difference at the system input (the difference between input and output) will be $1+\mathbf{g}\Phi(s)\mathbf{b}$. The necessary and sufficient condition for optimality of a gain vector **g** can be expressed using the return difference of the system only[2], which are,

(a) $P_k(s)$ satisfies the Routh-Hurwitz conditions, (b) $|1+\mathbf{g}\Phi(s)\mathbf{b}|^2 \geq 1,\ s=j\omega$ (3)

The condition (a) is related to the stability of the closed loop system and is automatically satisfied if the poles of the closed loop system are pseudo negative. The condition (b) is the classical criterion of optimality and it ensures that the closed loop system is less sensitive to variation in parameters of the system. The system will be just optimal if the absolute value of the return difference is unity and the degree of optimality will increase with increase in the absolute value of the return difference. We can also be express the return difference and the condition (b) in terms of P(s) and $P_k(s)$ as,

$$\text{(a) } 1+\mathbf{g}\Phi(s)\mathbf{b}=P_k(s)/P(s),\ \text{(b) } |P_k(s)/P(s)|^2 \geq 1,\ s=j\omega \quad (4)$$

If we set s=jω and plot Eq.(4b) on complex plane, the graphical interpretation of the optimality criterion is that a control law is optimal if and only if the complex plot of Eq.(4b) does not penetrate a unit disk centered around the origin· This has been shown in Fig.2 for optimal and not optimal cases.

(i) Single Degree of Freedom System: We will first show the region of optimal pole locations for a SDOF system and then show its extension to MDOF system. Consider a system with the closed and open loop poles as $-a\pm jb$ and $-a_c\pm jb_c$ and the respective polynomials as $P(s)=s^2+2as+(a^2+b^2)^2$ and $P_k(s)=s^2+2a_cs+(a_c^2+b_c^2)^2$ respectively. Substituting these polynomials into Eq.(4b), we can express the optimality condition in terms of pole locations as:

(i) $(a_c^2+b_c^2)^2 \geq (a^2+b^2)^2$, (ii) $a_c^2-b_c^2 \geq a^2-b^2$, and (iii) (i) and (ii) cannot have equal sign simultaneously. (5)

We plot these conditions for a structure with ω_s= 1 rad/sec and ζ_s(inherent damping)=1 % as shown in Fig.(3). Curves 1 and 2 in this plot correspond to{equality in (i), inequality in (ii)} and (inequality in (i), equality in (ii)} respectively. We note that curve 1 is a part of a circle with radius equal to ω_s and curve 2 approaches the 70.7 % damping line asymptotically. Now, if we consider inequality in both (i) and (ii) then we obtain the hatched area. This is the region of optimal pole locations for the control of a SDOF system.

(ii) Multi-Degree of Freedom: Suppose complex conjugate pairs of poles of the open-loop and closed loop system are $(\lambda_1,\tilde{\lambda}_1)$, $(\lambda_2,\tilde{\lambda}_2)$, $(\lambda_3,\tilde{\lambda}_3)..(\lambda_n,\tilde{\lambda}_n)$, and $(\rho_1,\tilde{\rho}_1)$, $(\rho_2,\tilde{\rho}_2)$, $(\rho_3,\tilde{\rho}_3)...(\rho_n,\tilde{\rho}_n)$ respectively. Then, the condition in Eq.(4b) can be rewritten as,

$$|P_k(s)/P(s)|^2=\prod_{i=1}^{n}|\psi_i(s)|^2\geq 1,\ \psi_i(s)=(s-\rho_i)(s-\tilde{\rho}_i)/(s-\lambda_i)(s-\tilde{\lambda}_i) \qquad (6)$$

Although this condition can be satisfied by infinite number of trajectories of closed loop poles, we can simplify this problem significantly if we place the poles inside the region of optimal pole location in Fig.1 for each of the poles to be moved. Placing the poles in this way, the optimality condition in Eq.(6) will always be satisfied because for any pole i, $|\psi_i(s)|^2\geq 1$ in the optimal region.

(B) Multi-Actuator System: The block diagram for the closed loop system has been shown in Fig.4. Although different return difference matrices(RDM) can be written by breaking the loop at different points, the determinants of all the RDM's will be equal. For example, breaking the loop at points 1 and 2, we get

$$\Delta(s)=\det[\mathbf{I}+\mathbf{G}\boldsymbol{\Phi}(s)\mathbf{B}]=\det[\mathbf{I}+\mathbf{R}^{1/2}\mathbf{G}\boldsymbol{\Phi}(s)\mathbf{B}\mathbf{R}^{-1/2}]=P_K(s)/P(s) \qquad (7)$$

respectively. Hence, similar to single-actuator case, the necessary condition of optimality is expressed as,

$$\Delta(j\omega)\Delta(-j\omega)=|\Delta(j\omega)|^2\geq 1 \qquad (8)$$

However, unlike single-actuator systems, this is not sufficient condition for optimality. This means that given a gain matrix **G**, even though the closed-loop system is asymptotically stable and the plot of Eq.(8) doesn't penetrate the unit disk, we cannot conclude that the gain matrix **G** is optimal. Conversely, if the complex plot penetrates the unit disk, then the **G** matrix is certainly not optimal. Using the argument similar to that of for MDOF with single-actuator, hatched area in Fig.3 can be used as optimal region for this case also. However, the optimal gain matrix will have to be calculated by minimizing a quadratic performance index using the **Q** matrix calculated by the inverse control method for the desired location of poles.

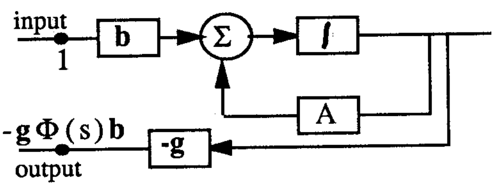

Fig. 1 Block diagram of a single-actuator system

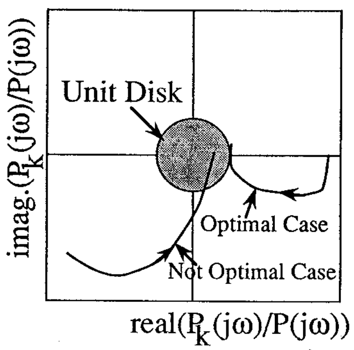

Fig. 2 Graphical representation of optimality criterion.

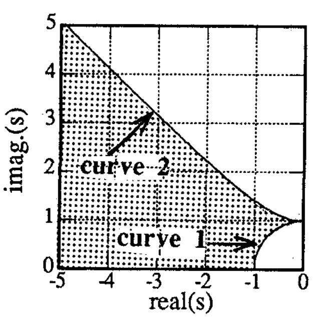

Fig. 3 Region of optimal pole locations for SDOF system with single actuator.

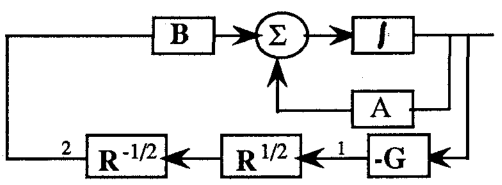

Fig. 4 Block diagram of a multi-actuator system ($RDM_1=[\mathbf{I}+\mathbf{G}\Phi(s)\mathbf{B}]$, $RDM_2=[\mathbf{I}+\mathbf{R}^{1/2}\mathbf{G}\Phi(s)\mathbf{B}\mathbf{R}^{-1/2}]$)

3. References

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