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# Precise Determination of Flutter by Multi-Mode Analysis

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## Introduction

This paper concerns the evaluation of coupled or "stiffness" driven flutter for Long-Span Bridges. Modern Long-Span Bridges exhibit complicated three dimensional vibrational characteristics. When we focus on the stiffness driven flutter, the two mode analysis may not provide accurate estimates and thus multi-modal or three dimensional flutter analysis is often employed.

With the example presented we aim at: 1) finding the fundamental mode combination; and 2) tracing accurately the target frequency and thus the velocity of flutter onset.

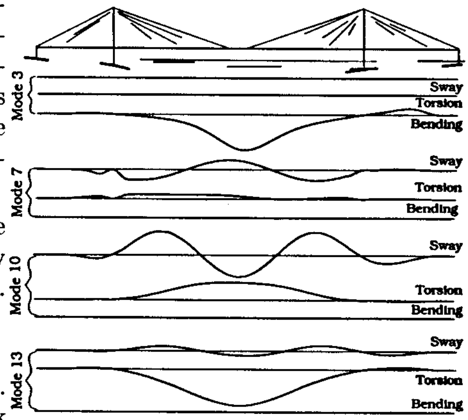


**Fig. 1:** Fundamental mode combination.

**Table 1:** Equivalent masses.

| No. | Bending $[ts^2/m^2]$ | Sway $[ts^2/m^2]$ | Torsion $[ts^2]$ |
|---|---|---|---|
| 3 | $0.200\times10^{+1}$ | - | - |
| 7 | - | $0.108\times10^{+2}$ | $0.649\times10^{+5}$ |
| 10 | - | $0.231\times10^{+1}$ | $0.191\times10^{+4}$ |
| 13 | - | $0.434\times10^{+2}$ | $0.109\times10^{+3}$ |

## Analysis and Discussion

Our example involves a Long-Span Cable-Stayed Bridge. The aerodynamic forces are evaluated for the bridge deck as for a flat plate [2]. Theoretical background of the 3D flutter analysis utilized can be found in [1] and [4]. As shown in Fig. 1, the 1st symmetric modes in heaving and in torsion are modes 3 and 13. Both possess minimum values of equivalent mass and mass moment of inertia (Tab. 1). This combination is usually regarded as a basic for the analysis of coupled flutter. Results from the present investigation are summarized in Tab. (2). In the case of multi-modal flutter analysis, we suggest that the number of modes should be continuously increased, until stable results are obtained. The trial shows that no drastic differences appear when more than 15 modes are included. The minimum onset velocity is $U_{ons}$ = 110.56 $m/s$ for the combination 1 to 15, target mode 7. The same combination with target modes 10 or 13 yields to higher onset velocities. For both cases, flutter appears in mode 7 at a lower velocities. Mode 7 has smaller then mode 13 mass in sway, but larger equivalent mass moment of inertia in torsion. These results are surprising because both modes 7 and 10 possess sway components and in the analysis aerodynamic sway forces have been neglected.

From Tab. (2) is evident that the fundamental mode combination should consist of modes 3,7,10,13. It this Table $U_r = U/f_{ons}B$, where $U$ is the wind velocity, $B$ is the deck width, and $f_{ons}$ is the natural frequency of the flutter state. The $U-\delta$ and $U-f$ diagrams are shown in Fig. 4, where $\delta$ is the total damping and $f$ is the current natural frequency.

It is crucial to recognize that since $\delta$ and $f$ paths are very close, mode 7 and 10, it is necessary to perform the calculation with a small velocity step (e.g. $\Delta U = 0.1\,m/s$). When the velocity increases in the analysis, the target frequency may or may not cross other frequencies. We argue that only this target mode has physical meaning because *a priori* is assumed that the aerodynamic forces are governed solely by its frequency. Thus switching to any other frequency leads to a distortion of the final result. When high velocity step $\Delta U$ is used, tracing of a false crossover can bring erroneous result (Fig. 2). One may find similar problems for the analysis traced in the Argand plane [1] and [5].

In order to find the mechanism allowing a initially "sway" mode to display the lowest flutter onset, we attempt to suppress it. Because little information is available the quasi-steady aerodynamic derivative $P_1^*$ is used [3]. When the target mode is 7 or 10, the onset remains almost the same while flutter does not occur in mode 13. The modal amplitudes around the flutter state, show that mode 7(10) couples with modes 3 and 13 while its sway diminishes (Fig. 3). Whereas, mode 13 couples with 7 and 10 taking their sway components while its torsional component scales down. Thus the $U-\delta$ path of mode 7 turns sharply down while for mode 13, it becomes flat.

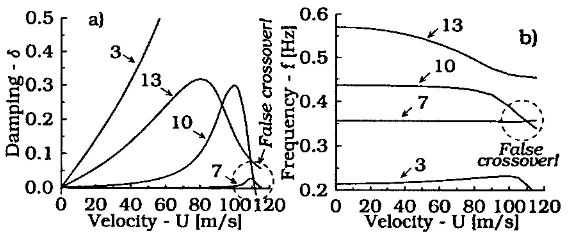


**Fig. 2:** $U-\delta$ and $U-f$ diagrams, $\Delta U = 10\,m/s$.

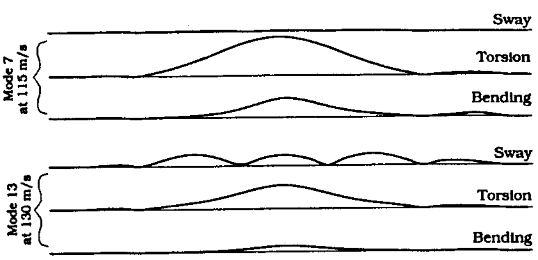


**Fig. 3:** Amplitudes for trial modes 7 and 13.

**Table 2:** Mode combination and flutter onset.

| Mode | Case | $U_{ons}$ $[m/s]$ | $U_r$ $[-]$ | $f_{ons}$ $[Hz]$ |
|---|---|---|---|---|
| 7 | 3,7 | — | - | - |
| 10 | 3,10 | 150.30 | 12.082 | 0.4147 |
| 13 | 3,13 | 118.37 | 10.957 | 0.3601 |
| 7 | 3,7,10,13 | 111.03 | 10.651 | 0.3475 |
| 10 | 3,7,10,13 | 111.78 | 10.760 | 0.3463 |
| 13 | 3,7,10,13 | 159.02 | 11.821 | 0.4484 |
| 13† | 3,7,10,13 | — | - | - |
| 7 | 1 to 15 | **110.56** | **10.466** | **0.3521** |
| 7 | 1 to 20 | 110.82 | 10.500 | 0.3518 |
| 7 | 1 to 30 | 110.62 | 10.481 | 0.3518 |
| 7 | 1 to 40 | 110.70 | 10.491 | 0.3517 |

a) The deck is $B=30\,m$; $\rho_{air}=0.125\,kg.s^2/m^4$;
b) By Selberg formula $U_{ons}^{3,7} = 183.84\ m/s$, $U_{ons}^{3,10} = 167.77\ m/s$, and $U_{ons}^{3,13} = 123.04\ m/s$;
c) The "—" means no flutter for $U=0 \sim 1000\,m/s$;
d) The † denotes the sway damping $P_1^*$ is included.

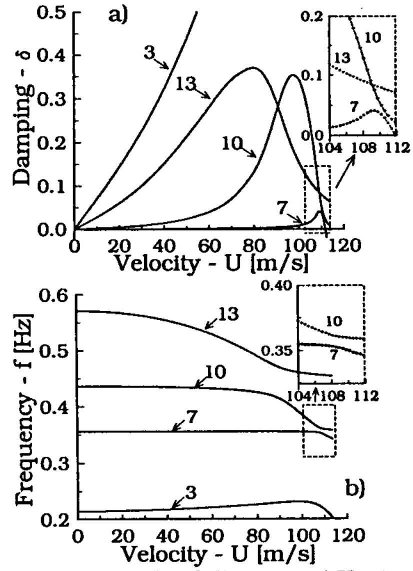


**Fig. 4:** $U-\delta$ and $U-f$ diagrams, $\Delta U = 0.1\,m/s$.

As final remarks, we recommend: 1) to use small velocity step around the complex frequency crossovers; and 2) to trace the solution by $U-\delta$ and $U-f$ diagrams instead of the Argand plotting.

## Acknowledgement

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