I - 571 AUTOREGRESSIVE SIMULATION OF WIND LOADING

Jovito C. SANTOS Student, Yokohama National University
Hitoshi YAMADA Member, Yokohama National University
Toshio MIYATA Member, Yokohama National University

Introduction

Very flexible structures exhibit complicated behavior under various load excitations. The action of wind presents complexities in the wind-structure interaction which is further aggravated when the wind characteristics change along the structure. In terms of computational effort, it is more convenient to perform the dynamic analysis of structures in the frequency domain. However, for cases where system nonlinearity and coupling between motion is expected, the time domain approach becomes an alternative method. In order to perform the dynamic analysis in the time domain, it is necessary to express the loading excitations as a temporal or spatial function or both.

For long span bridges, wind characteristics vary along the span so that the time-space variation has to be considered in the analysis. The fluctuating velocities in wind is normally described by its spectral density and can be taken as a random process. One method to generate the time history of wind or forces is to use the superposition of trigonometric functions, i.e. cosine functions with random phase angles. Such method requires much computational effort for multi-dimensional process especially when the generated series becomes very long. On the other hand, the use of recursive digital filtering method to simulate the process greatly enhance computational efficiency. This paper considers the use of multi-dimensional autoregressive model (AR) to generate the time series of fluctuating wind and forces.

Autoregressive approximation

In the autoregressive model, say of order p , the current observation u(t) of the process is expressed as a finite, linear aggregate of the previous values of the process - going back p periods, together with a random disturbance in the current period. For an n - variate autoregressive (AR) process U(t) of order M, the t^{th} sample can be generated from the previous ones using the relation

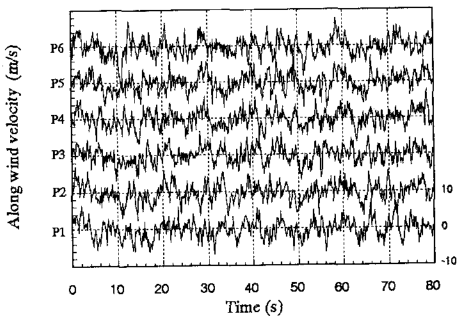
$$U(t) = \sum_{m=1}^{M} A(m)\ U(t - m\Delta t) + N(t) \qquad (1)$$

where $U(t) = [\, u_1(t)\ u_2(t)\ \ldots\ u_n(t)]^T$, A(m) is an n x n matrix of AR coefficients, Δt is the time interval, $N(t) = [\varepsilon_1(t)\ \varepsilon_2(t)\ \ldots\ \varepsilon_n(t)]^T$ which is a white noise, and M is the order of AR model.

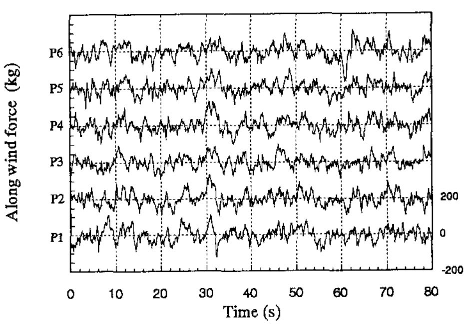
The primary concern would be to determine the A(m) coefficients based on the prescribed correlation function. Post multiplying Equation (1) by $U(t - k\Delta t)$ and assuming that the white noise and the series are uncorrelated at some time lag (i.e. $R_{NU}(-k) = 0$ for k > 0), the following result is obtained

$$R^T_{UU}(k) = -\sum_{m=1}^{M} A(m) R_{UU}(m-k) \qquad ,k = 1,2,\ldots M \qquad (2)$$

Since the autocorrelation function $R_{UU}(k)$ is known, the A(m) coefficients can be determined. Akaike [1] presented a method to determine the coefficients A(m) by minimizing the error $\overline{\varepsilon^2(t)}$.



(a) simulated fluctuating velocities



(b) simulated fluctuating force

Figure 1 Simulated time histories

Wind and force spectra

The autoregressive approximation requires a prior knowledge of the auto and cross correlation functions to determine the coefficients A(m). For wind loads, the fluctuating part of wind velocities are expressed as spectral density functions S(n) and using the inverse fourier transform, the correlation function can be obtained as

$$R(\tau) = \int_{-\infty}^{\infty} S(n) e^{i\omega\tau} dn \qquad (3)$$

The u - component of the velocity fluctuations given by the Hino spectrum was used in this study, expressed as

$$\frac{nS_u(n)}{\overline{u}^2} = 0.4751 \frac{n}{n'} \left\{ 1 + \left(\frac{n}{n'} \right)^2 \right\}^{-\frac{5}{6}} \qquad (4)$$

where $n' = 1.718 x 10^{-2} \frac{\alpha k_r \overline{U}_{10}}{I_u^3} \left(\frac{z}{10} \right)^{(2m-3)\alpha - 1}$

For the fluctuating part of the wind force directly attributable to turbulence, say drag, the force spectrum is given as

$$S_{FD}(n) = \rho^2 \overline{U}^2 B^4 C_D^2 S_u(n) \chi_D^2(n) \qquad (5)$$

where the aerodynamic admittance as suggested by Holmes [2] takes the form

$$\chi_D^2(n) = \frac{1}{1 + 4\left(\frac{nB}{\overline{U}} \right)} \qquad (6)$$

The cross - spectra between records at different locations can be expressed by its coherence function. Between points $p_1(y_1,z_1)$ and $p_2(y_2,z_2)$ the coherence function is assumed to take the form [3],

$$Coh(r,n) = e^{-\hat{f}} \qquad (7)$$

where $\hat{f} = \frac{n\left[C_z^2 (z_1 - z_2)^2 + C_y^2 (y_1 - y_2)^2 \right]^{\frac{1}{2}}}{\frac{1}{2}\left[\overline{U}_{z_1} + \overline{U}_{z_2} \right]}$

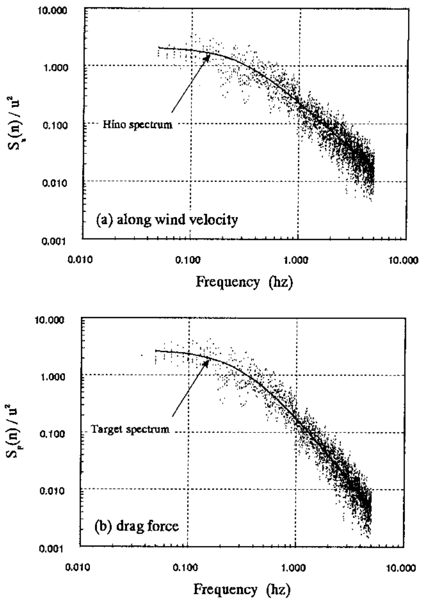


Figure 2 Spectra of simulated series

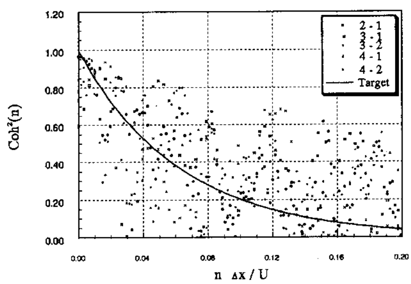


Figure 3 Coherence of simulated series

Simulation of wind and forces

Using the AR process defined by Equation (1), a multidimensional time series was generated for the wind and force spectra described above. The prescribed auto spectra and cross spectra obtained using the coherence function was used to determine the correlation functions using inverse fourier transform. The Akaike method was used to determine the autocorrelation coefficients A(m). Figure 1 shows the generated time histories of the fluctuating wind velocities and forces at different points. Close correlation between time histories at adjacent points can be seen from the figure. Figure 2 shows the spectra of the generated series of order AR(100) superimposed with the target spectra. The coherence functions between different points are shown in Figure 3. It should be noted that a sufficient order p of the AR(p) process is needed to approach the target spectra.

Concluding remarks

Since the time domain approach to dynamic analysis requires the time history of forces, efficient methods to simulate such time series is indicated. In this study, the multidimensional simulation of wind and forces was performed using the autoregressive (AR) model. The model is developed based on the prescribed spectral description at one point and the cross-spectra between points at different locations.

References

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[3] Simiu,E. & Scanlan,R.H. *Wind Effects on Structures 2Ed.*,John Wiley & Sons,1986.