IV-38 ANALYSIS OF RHYTHMICAL UNSYNCHRONISED TIME SPACE DIAGRAM BY LINEAR PROGRAMMING

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1. INTRODUCTION

Time space diagram is known, in many cases, to be more suitable than bar charts and network methods, particularly for planning, scheduling and monitoring of projects which are characterised by repetitive operations that are performed at all stages of the projects. Examples of such projects are highway, pipeline and high-rise building construction. Time space diagram can be formulated as a linear programming problem, so as to obtain the start times and duration of various activities and the minimum completion time of a project. However, application of linear programming algorithm to time space diagram has not been tried up to date. In this paper the formulation of the rhythmical unsynchronised time space diagram as LP problem is proposed and its application to a small pipeline construction project is shown. Particularly of great significance in this paper is that the analysis will provide information such as earliest start time and duration of the various activities and the minimum duration of the project. Rhythmical unsynchronised time space diagram can be defined as time space diagram showing two or more activities occurring regularly in sequence, the rate of output for each activity may be different but it is assumed that the rate of output for each activity is uniform throughout the project.

2. FORMULATION OF THE LP ALGORITHM

Fig.1 is a rhythmical unsynchronised time space diagram showing activities i and i+1. To avoid interruptions two kinds of constraints should be considered, time and space constraints. These are shown in Fig.1 as ①, ②, ③ and ④. In addition to these constraints, for every activity there are minimum and maximum attainable rates of output depending on choice of method or equipment.

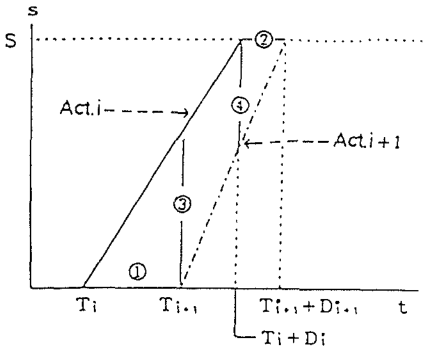

Fig.1 Rhythmical unsynchronised time space diagram

A) To avoid time interruptions (for i= 1,2,...,N-1)

① $T_{i+1} - T_i \geq BT_i$

② $T_{i+1} + D_{i+1} - (T_i + D_i) \geq BT_i$

$\therefore T_{i+1} - T_i + D_{i+1} - D_i \geq BT_i$

where BT_i = minimum buffer time between activities i and i+1
T_i = earliest start time of activity i
D_i = duration of activity i
N= number of activities of the project

B) To avoid space interruptions (for i=1,2,...,N-1)

③ $\frac{S}{Di}(T_{i+1}-T_i) \geq BS_i$

$\therefore S \cdot T_{i+1} - S \cdot T_i - BS_i \cdot D_i \geq 0$

④ $S - \frac{S}{D_{i+1}}\{(T_i + D_i) - T_{i+1}\} \geq BS_i$

$\therefore S \cdot T_{i+1} - S \cdot T_i + (S - BS_i) \cdot D_{i+1} - S \cdot D_i \geq 0$

where BS_i = minimum buffer space between activities i and i+1
S= total length of the project

C) Duration constraints (for i=1,2,...,N)

⑤ $D_i^{min} \leq D_i \leq D_i^{max}$

where D_i^{min} = minimum duration of activity i
D_i^{max}= maximum duration of activity i

D) The objective function is given by

$$FT = \sum_{i=1}^{N-1} (T_{i+1} - T_i) + D_N \longrightarrow min$$

where FT= duration of the project

E) The number of constraints and decision variables varies with the number of activities. It is given by the equation:
Total number of constraints = 2N+4(N-1) = 6N-4

3.APPLICATION TO PIPELINE CONSTRUCTION PROJECT

It is not difficult to formulate all constraints as well as objective functions of LP for time space diagram for pipeline construction work, knowing the number of activities, total length of project, minimum buffer times and spaces between subsequent activities and the minimum and maximum attainable rate of output for the various activities. Fig.2 shows a print out from a computer program which formulates all the constraints automatically. In this example the total length of pipeline to be constructed is 120 metres and the project consists of four activities. To run the program the computer prompts for the number of activities, the total length of the project, the maximum and minimum attainable rate of output for the various activities and the minimum buffer space and time between successive activities. The resultant LP can be solved using two-phase simplex method.

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INPUT DATA
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NUMBER OF ACTIVITIES IS ( 4 )

TOTAL LENGTH OF PROJECT IS ( 120 )

MINIMUM RATE OF ACTIVITY 1 RMIN( 1 ) 3
MAXIMUM RATE OF ACTIVITY 1 RMAX( 1 ) 4
MINIMUM RATE OF ACTIVITY 2 RMIN( 2 ) 2
MAXIMUM RATE OF ACTIVITY 2 RMAX( 2 ) 5
MINIMUM RATE OF ACTIVITY 3 RMIN( 3 ) 2
MAXIMUM RATE OF ACTIVITY 3 RMAX( 3 ) 6
MINIMUM RATE OF ACTIVITY 4 RMIN( 4 ) 1
MAXIMUM RATE OF ACTIVITY 4 RMAX( 4 ) 4

MINIMUM BUFFER SPACE BETWEEN ACTIVITY 1 AND 2 BS( 1 ) 9
MINIMUM BUFFER TIME BETWEEN ACTIVITY 1 AND 2 BT( 1 ) 3
MINIMUM BUFFER SPACE BETWEEN ACTIVITY 2 AND 3 BS( 2 ) 6
MINIMUM BUFFER TIME BETWEEN ACTIVITY 2 AND 3 BT( 2 ) 4
MINIMUM BUFFER SPACE BETWEEN ACTIVITY 3 AND 4 BS( 3 ) 6
MINIMUM BUFFER TIME BETWEEN ACTIVITY 3 AND 4 BT( 3 ) 2

***END OF INPUT DATA***

THE NUMBER OF CONSTRAINTS IS EQUAL TO ( 20 )

CONSTRAINTS
***********

T( 2 )-T( 1 )>= 3
T( 3 )-T( 2 )>= 4
T( 4 )-T( 3 )>= 2
T( 2 )-T( 1 )+D( 2 )-D( 1 )>= 3
T( 3 )-T( 2 )+D( 3 )-D( 2 )>= 4
T( 4 )-T( 3 )+D( 4 )-D( 3 )>= 2
 120 *T( 2 )- 120 *T( 1 )- 9 *D( 1 )>=0
 120 *T( 3 )- 120 *T( 2 )- 6 *D( 2 )>=0
 120 *T( 4 )- 120 *T( 3 )- 6 *D( 3 )>=0
 120 *T( 2 )- 120 *T( 1 )+( 111 )*D( 2 )- 120 *D( 1 )>=0
 120 *T( 3 )- 120 *T( 2 )+( 114 )*D( 3 )- 120 *D( 2 )>=0
 120 *T( 4 )- 120 *T( 3 )+( 114 )*D( 4 )- 120 *D( 3 )>=0
D( 1 )>= 30
D( 1 )<= 40
D( 2 )>= 24
D( 2 )<= 60
D( 3 )>= 20
D( 3 )<= 60
D( 4 )>= 30
D( 4 )<= 120

OBJECTIVE FUNCTION
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MINIMUM DURATION OF PROJECT (FT)=T( 4 )-T( 1 )+D( 4 )
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Fig.2 Example of computer output

4.CONCLUSION

This paper has shown that time space diagram can be formulated and easily solved by LP. The importance of this analysis include scheduling of the activities, choice of equipment and methods and resource allocation to achieve minimum project duration.