III－167 Calculation of Softening Based on Peak Strength Variation with State Parameter

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Introduction

It is accepted in geotechnical engineering that the peak strength of a soil is variable. The general tendency is that the peak strength decreases with the increase of void ratio and with the increase of mean stress level. In their research, the authors (Liu & Nakai 1992) have concluded that the variation of the peak strength with void ratio and the stress level leads to peak strength state unstable. Consequently, for some disturbances, the soil element at peak strength state either collapses or softens. The final strength of a sand also seems to be dependent on mineralogy only. Therefore, if the deviation of peak strength of soil from the final strength is considered in a constitutive model, the internal principle of soil requires the model to describe the peak strength as a soil strain state and stress state dependent quantity, and to describe the decrease of the strength from peak value to the finial value after the peak strength is reached, i,e., the softening of soil. Otherwise, the use of soil peak strength suggested in the model will result in overestimate of the capacity of soil resistance to failure under large deformation and possibly result in underestimate of the associated deformation; and the instable state of soil is wrongly interpreted as stable. In this paper a simple constitutive equation is proposed. The model is introduced as demonstration (1) to describe the influence of void ratio and mean stress level on peak strength, (2) to indicate the unstable state of soil after the peak strength is reached; (3) and to predict softening of soil from observing soil behaviour during pre–peak strength state.

t_{ij} Concept and Variables Used

The simple model is based on SMP theory introduced by Matsuoka and Nakai[1] and t_{ij} concept introduced by Nakai[2] and the state parameter Φ introduced by Been at al[3]. There are the variables used and the detailed explanation can be found from the references:

X : stress ratio based on SMP; t_{ij} : a modified stress tensor;
X_f : final strength for a soil; Φ : state parameter;
X_p : peak strength; p : mean stress level;
γ_{SMP}^* : distortional strain quantity based on SMP; ε_v : volumetric strain.

A Constitutive Equation for Soil Behaviour under Monotonic Virgin Loading in the Stress Tensor Space

Two postulates

(1) For a given soil element under virgin monotonic loading there is a one to one relationship between stress ratio and the distortional strain γ_{SMP}^*.

(2) A linear relationship between the peak strength and the state parameter exists for soil. Hence,

$$X_p=X_f\,(1+\mu\Phi) \tag{1}$$

The simple constitutive relation

The total strain is divided into two parts: the elastic part $d\varepsilon_{ij}^e$ and the stress ratio effect part $d\varepsilon_{ij}^x$. $d\varepsilon_{ij}^e$ is computed by Hook's law with a zero value for Possion's ratio. Thus,

$$d\varepsilon_{ij}^e=k\;d\sigma_{ij}\;/\;p \tag{2}$$

For $d\varepsilon_{ij}^x$, the distortional strain γ_{SMP}^* is firstly computed from the following formula,

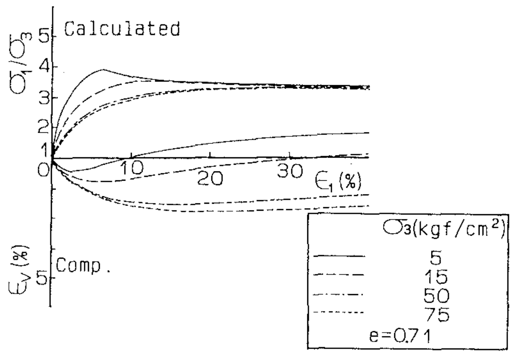

Fig.1. Soil behaviour under different confining pressures calculated

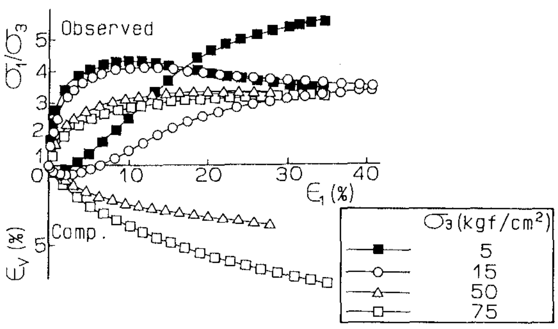

Fig.2. Soil behaviour under different confining pressures observed

$$\gamma^*_{SMP}=\sum_{i=0}^{m} C_i \left[\frac{\arcsin\frac{X}{X_p}}{e^{a\Phi+X_p-X}-1}\right]^i \tag{3}$$

To demonstrate the working of a simple model, only the first item is used in this paper. Thus

$$\gamma^*_{SMP}=\frac{b\arcsin\frac{X}{X_p}}{e^{a\Phi+X_p-X}-1} \tag{4}$$

The volumetric strain is then calculated from the following stress ratio and strain increment ratio relation.

$$\frac{d\varepsilon_v}{d\gamma^*_{SMP}}=\frac{X_f-X}{\beta} \tag{5}$$

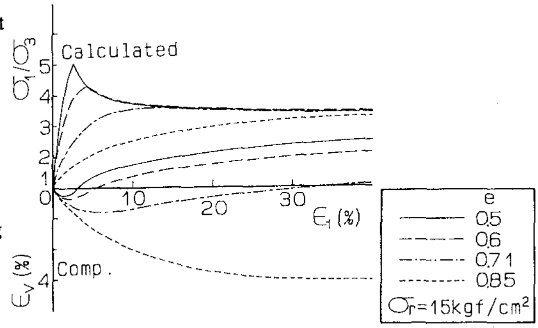

Fig.3. Soil behaviour under different initial void ratios calculated

The components for the strain tensor increment can be found based on a formula suggested by Liu[5)]

$$d\varepsilon^X_{ij}=\Lambda\left(\sigma_{ij}-\frac{P}{3}\delta_{ij}\right)+\frac{d\varepsilon_v}{3}\delta_{ij} \tag{6}$$

Based on the above six formula, the strain path corresponding to any virgin monotonic stress path can be computed. μ, k, C_i, a, b and β are soil parameters. As the main equation, expression (4) the distortion strain and stress ratio relationship, is expressed by a single formula, softening can be predicted by observing the behaviour of soil during pre-peak strength state. Because of the difficulty in obtaining satisfactory data for softening, extrapolation of soil softening behaviour is of practical convenience. The possibility of the extrapolation is also logically necessary, because it is unlikely that the principle which governs the softening has zero-influence on soil behaviour before the peak strength is reached. Liu[5)] found that the principle governing softening of soil generally does have influence on soil behaviour in so called hardening behaviour range. Consequently, softening can be studied from soil behaviour before peak strength is reached.

Values for the Parameters and the Calculation and Comments

To demonstrate the working of the constitutive equation, some calculations are made. The following values for the parameters are chosen for Toyoura sand (further improvement on the calculation is possible through the optimization of values for the parameters), τ=0.86, λ=0.041, X_f=0.589, β=0.7, k=0.006, a=0.2, b=0.01, μ=1.8.

For the calculation, the stress path follows a conventional triaxial compression test. Samples are firstly isotropically consolidated to a chosen cell pressure; after that, the samples are sheared at constant cell pressure. Two sets of calculations are made. Set I, different values for cell pressures are chosen; they are: σ_r=5kgf/cm^2, 15kgf/cm^2, 50kgf/cm^2 and 75kgf/cm^2. All the samples have the same initial void ratio as 0.71 at the start of consolidation process. The calculation curves are shown in Fig.1. Some test results which follow the same stress and strain constraint are shown in Fig.2. The details for the tests can be found from Liu and Nakai (1992). Set II, different values of initial void ratios are chosen for the samples; they are: e=0.5, 0.6, 0.71, and 0.85. The cell pressure is fixed for all the test as σ_r=15kgf/cm^2. The calculation curves are shown in Fig.3.

It is seen that the equation describes the influence of stress level and void ratio on the type of soil behaviour. The equation describes the influence of state parameter on peak strength and the stiffness of soil response to loading. Examining the equation, the following characteristics are seen. For sand with a negative Φ, the sand hardens steadily until failure is reached at theoretically infinitive large deformation. For sand with a positive Φ, stress ratio X has a maximum value X_p at a definitive deformation. As the deformation further increases, X decreases because Φ decreases resulting from the volumetric expansion. X decreases continuously until it has the same value as the final strength. This state occurs, theoretically, at infinitive large deformation. From the physical point of view, the decrease of X after the peak strength is reached indicates that the peak strength state is unstable.

It is seen from the calculation that the softening curves X: ε_1 for a soil is fixed. There is a one-to-one relationship between γ_{SMP}^* and Φ or between X and γ_{SMP}^*, if X=X_p (formula (4)).

References

1) Matsuoka H. & Nakai T. (1974), Proc. JSCE, No.232, pp.59–70. 2) Nakai T. & Matsuoak H. (1986), Soils and Foundations, Vol.26, No.3, 81–98. 3) Been K. & Jefteries M. G. (1985), Geotechnique, Vol.35, No.1, 99–112. 4) Liu D. F. and Nakai T. (1992), Proc. 27th annual meeting of JSSMFE. 5) Liu D. F. (1991), Ph. D. thesis, Glasgow University.