II-575 NUMERICAL ANALYSIS FOR KINEMATIC UNSTEADY FLOWS IN MOUNTAINOUS STREAM NETWORKS

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1. Introduction

A formulation for stream flow analysis based on the kinematic wave equation and the weighted residual method is presented. The resulting four point equation is analysed and compared with the Muskingum method.

2. Discretization by The Weighted Residual Method

The basic equations for kinematic unsteady flow are,

continuity equation : $\partial Q / \partial x + \partial A / \partial t = q$ (1)

momentum equation : $A = \gamma Q^{P}$ (2)

Q : discharge, A : cross sectional area
t : time, x : distance, q : lateral inflow per unit length, γ, P: constants

Considering a triangular cross sectional channel as in Fig-1,

$\gamma = (n/\sqrt{\sin\theta})^{3/4} \times (8/\sin\delta)^{1/4}$ and $P=3/4$

n:Manning's coefficient θ:channel slope
δ:angle between wetted sides of channel

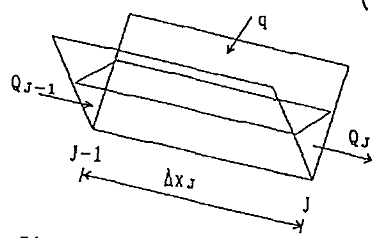


Fig-1 Defintion sketch

Substituting equation (2) into equation (1) gives equation (3),

$$\partial Q / \partial x + \lambda \partial Q / \partial t = q \;, \quad \lambda \equiv dA/dQ = \gamma P Q^{P-1} \tag{3}$$

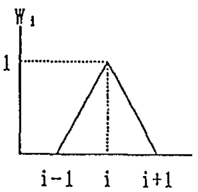
Discretizing equation (3) using a shape function as in Fig-2 and a weighting function as in Fig-3,

$$TQ + S dQ/dt - \Phi = 0 \tag{4}$$

where

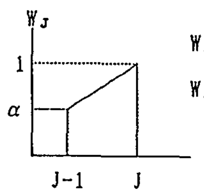
$$S = \begin{vmatrix} \langle\lambda\rangle_1^2(\Delta x_2/6+\alpha\Delta x_2/3) & \langle\lambda\rangle_1^2(\Delta x_2/3+\alpha\Delta x_2/6) & & & & \\ & \ddots & & & & \\ & & \langle\lambda\rangle_{J-1}^J(\Delta x_J/6+\alpha\Delta x_J/3) & \langle\lambda\rangle_{J-1}^J(\Delta x_J/3+\alpha\Delta x_J/6) & & \\ & & & \ddots & & \\ & & & & \langle\lambda\rangle_{m-1}^m(\Delta x_m/6+\alpha\Delta x_m/3) & \langle\lambda\rangle_{m-1}^m(\Delta x_m/3+\alpha\Delta x_m/6) \end{vmatrix}$$

$$T = \begin{vmatrix} (-1/2-\alpha/2) & (1/2+\alpha/2) & & \\ & \ddots & \ddots & \\ & & (-1/2-\alpha/2) & (1/2+\alpha/2) \end{vmatrix} \qquad \Phi = \begin{vmatrix} \langle q\rangle_1^2(\Delta x_2/2+\alpha\Delta x_2/2) \\ \vdots \\ \langle q\rangle_{m-1}^m(\Delta x_m/2+\alpha\Delta x_m/2) \end{vmatrix}$$



$W_I = (x - x_{I-1})/\Delta x_I \quad (x_{I-1} \leq x \leq x_I)$
$W_I = (x_{I+1} - x)/\Delta x_{I+1} \quad (x_I \leq x \leq x_{I+1})$
$W_I = 0 \quad (x < x_{I-1},\ x_{I+1} < x)$

Fig-2 Shape function



$W_J = (1-\alpha) X (x - x_{J-1})/\Delta x_J + \alpha \quad (x_{J-1} \leq x \leq x_J)$
$W_J = 0 \quad (x < x_{J-1},\ x_J < x)$
$0 \leq \alpha \leq 1$

Fig-3 Weighting function

Here $\langle\lambda\rangle_{J-1}^{J}$ and $\langle q\rangle_{J-1}^{J}$ indicate average values for the subreach having cross sections J-1 and J.

Generally as time steps n and n+1 are considered in equation (4),

$$\left.\begin{array}{ll}\text{for the first term of equation (4),} & Q = \beta Q^{n+1} + (1-\beta)Q^{n} \\ \text{for the second term of equation (4),} & dQ/dt = (Q^{n+1} - Q^{n})/\Delta t\end{array}\right\} \quad (5)$$

β : weighting factor in relation to time

Substituting equation (5) into equation (4) will give equation (6)

$$[\langle\lambda\rangle^{J}_{J-1}(\Delta x_J/\Delta t)(1/6+\alpha/3)+(-1/2-\alpha/2)]Q^{n+1}_{J-1} + [\langle\lambda\rangle^{J}_{J-1}(\Delta x_J/\Delta t)(1/3+\alpha/6)+\beta(1/2+\alpha/2)]Q^{n+1}_{J} = [\langle\lambda\rangle^{J}_{J-1}(\Delta x_J/\Delta t)(1/6+\alpha/3)-(1-\beta)(-1/2-\alpha/2)]Q^{n}_{J-1} + [\langle\lambda\rangle^{J}_{J-1}(\Delta x_J/\Delta t)(1/3+\alpha/6)-(1-\beta)(1/2+\alpha/2)]Q^{n}_{J} + \langle q\rangle^{J}_{J-1}\Delta x_J(1/2+\alpha/2) \quad (6)$$

Equation (6) is rewritten as equation (7),

$$A_{J-1}Q^{n+1}_{J-1} + B_J Q^{n+1}_{J} = C_{J-1} \qquad (J = 2,\ldots\ldots,m) \quad (7)$$

Expressing it in the matrix form, the unknown vector X could be found as follows,

$$X = N^{-1}(R-MX_1) \quad (8)$$

$$M = \begin{vmatrix} A_1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{vmatrix} \qquad N = \begin{vmatrix} B_2 & & \\ \cdot & & \\ A_{J-1}B_J & & \\ & \cdot & \\ & & A_{m-1}B_m \end{vmatrix} \qquad X = \begin{vmatrix} Q^{n+1}_2 \\ \cdot \\ \cdot \\ \cdot \\ Q^{n+1}_m \end{vmatrix} \qquad R = \begin{vmatrix} C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_{m-1} \end{vmatrix} \qquad X_1 = [Q^{n+1}_1]$$

For an initial value of $\langle\lambda\rangle^{J}_{J-1}$ an approximation as in equation (9) could be made and thereafter an iteration process could be carried out to obtain accurate results.

$$\langle\lambda\rangle^{J}_{J-1} = \langle\lambda(Q^{n+1})\rangle^{J}_{J-1} \simeq \langle\lambda(Q^{n})\rangle^{J}_{J-1} \quad (9)$$

3. Relationship Between Parameters of The Weighting Function And The Muskingum Model

$$\text{If} \quad [1+\alpha/(1+\alpha)]/3 = \theta \quad (10)$$

equation (6) could be rewriten as equation (11).

$$[\beta(Q^{n+1}_{J}-Q^{n+1}_{J-1})+(1-\beta)(Q^{n}_{J}-Q^{n}_{J-1})]/\Delta X_J + (\langle\lambda\rangle^{J}_{J-1}/\Delta t)[\theta(Q^{n+1}_{J-1}-Q^{n}_{J-1}) + (1-\theta)(Q^{n+1}_{J}-Q^{n}_{J})] = \langle q\rangle^{J}_{J-1} \quad (11)$$

Moreover for $\beta=1/2$, equation (12) is reduced to

$$Q^{n+1}_{J} = C_1 Q^{n}_{J-1} + C_2 Q^{n+1}_{J-1} + C_3 Q^{n}_{J} + C_o\langle q\rangle^{J}_{J-1}\Delta X_J \quad (12)$$

where $C_1 = (2K\theta+\Delta t)/(2K(1-\theta)+\Delta t)$ $\quad C_2 = (-2K\theta+\Delta t)/(2K(1-\theta)+\Delta t)$

$C_3 = (2K(1-\theta)-\Delta t)/(2K(1-\theta)+\Delta t)$ $\quad C_o = (2\Delta t)/(2K(1-\theta)+\Delta t)$

$K = \Delta x_J/\langle\lambda\rangle^{J}_{J-1}$, and $C_1 + C_2 + C_3 = 1$

It is clear that equation (12) is identical to the conventional Muskingum-Cunge equation. As $0<\alpha<1$, from equation (10) it could be clarified that the valid range of parameter θ is between 1/3 and 1/2.

4. Conclusion

A 4- point equation is developed for kinematic stream flow routing. A new range for the parameter θ of the Muskingum equation is proposed.

Reference

1)Weinmann,P. and Laurenson,E., Approximate flood routing methods, Jou. of Hyd.,Vol.105,HY12,1979,pp.1521-1537.

2)Holden,A. and Stephenson,D., Improved 4 point solution of the Kinematic equation, Jou. of Hyd.,Vol.26,NO.4,1988,pp.413-423.