II-570

Lateral variation of the vertically-averaged velocity in a compound channel flow

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INTRODUCTION

Recently studies on compound open-channel flows have progressed rapidly, both in laboratory investigations (see for example, the British SERC facilities at Wallingford [1]) and in numerical modelling, revealing thus more and more details about the flow structures and gaining better insights into the problems. The experimental data gathered from large scale physical models, of which the hydraulic parameters cover wide ranges, should also improve the performance of numerical modeling. The progresses are particularly welcome for solving such problems as the prediction of local bed deformations in a compound channel.

While all these are very important and should be pursued in the future, the engineer in practical engineering may still be intrigued by some simple, cheap and quick solutions to calculate the velocity and the discharge on the flood plain and in the main channel, and sum them up as the cross-sectional mean values. In the following a simple momentum analysis shall be applied to a compound channel flow in order to determine the lateral variation of the vertically-averaged velocity.

ANALYSIS

Considering the lateral momentum exchange in a compound open-channel flow, the momentum equations for the flow on the flood plain and the one in the main channel can be written, in a similar way as done by Xie [3], as:

$$\frac{\tau_{bf}}{D_f} - \frac{d\tau_f}{dz} = \rho g S$$
$$\frac{\tau_{bm}}{D_m} - \frac{d\tau_m}{dz} = \rho g S \qquad (1)$$

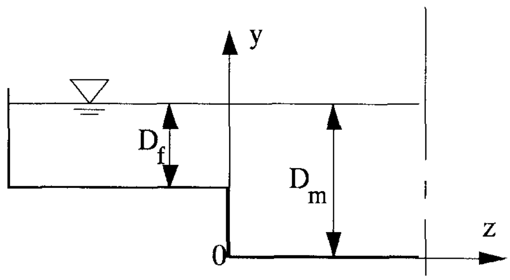

Fig.1 Definition sketch

where: the subscripts "m" and "f" designate flow properties in the main channel and on the flood plain, respectively; τ_b, the bottom shear stress on the flood plain or in the main channel; τ, the shear stress produced by the lateral momentum exchange; D, the water depth; ρ, the fluid density; S, the slope; z, the lateral coordinate (see Fig.1). On the other hand:

$$\tau_{bf} = \frac{f_f}{8}\rho V_f^2 \; ; \qquad \tau_{bm} = \frac{f_m}{8}\rho V_m^2$$
$$\rho g S = \frac{\rho f_f}{8D_f} U_f^2 \; ; \qquad \rho g S = \frac{\rho f_m}{8D_m} U_m^2 \qquad (2)$$

where: f is the friction coefficient; U, the mean velocity for the main channel or the flood plain; V, the vertically-averaged velocity, which varies in the lateral direction. Consequently Eq.1 can be rewritten as:

$$\frac{\rho f_f}{8D_f}(V_f^2 - U_f^2) - \frac{d\tau_f}{dz} = 0$$
$$\frac{\rho f_m}{8D_m}(V_m^2 - U_m^2) - \frac{d\tau_m}{dz} = 0 \qquad (3)$$

In order to obtain an analytical solution (see the following), let us assume that:

$$\tau_f = \rho\vartheta_f D_f V_f \frac{dV_f}{dz} = \frac{\rho\vartheta_f D_f}{2}\frac{dV_f^2}{dz}$$
$$\tau_m = \rho\vartheta_m D_m V_m \frac{dV_m}{dz} = \frac{\rho\vartheta_m D_m}{2}\frac{dV_m^2}{dz} \tag{4}$$

Using experimental results [1, 2], one can deduce an empirical relation for the coefficients ϑ_f and ϑ_m, as:

$$\frac{\vartheta_f}{\vartheta_m} = \frac{(2D_r)^{-4}}{D_r}\frac{V_m}{V_f}\ ; \qquad D_r = \frac{D_f}{D_m} \tag{5}$$

Substituting Eq.4 into Eq.3, one obtains:

$$a^2(V_f^2 - U_f^2) - \frac{d^2(V_f^2 - U_f^2)}{dz^2} = 0\ ; \qquad a^2 = \frac{f_f}{4\vartheta_f D_f^2}$$
$$b^2(V_m^2 - U_m^2) - \frac{d^2(V_m^2 - U_m^2)}{dz^2} = 0\ ; \qquad b^2 = \frac{f_m}{4\vartheta_m D_m^2} \tag{6}$$

The boundary conditions are:

1) for $z = -\infty$, $V_f = U_f$; for $z = \infty$, $V_m = U_m$;

2) for $z = 0$, $V_f = V_m$ and $\tau_f = \tau_m$

The analytical solutions for Eq.6 can be subsequently deduced as:

$$V_f = \left[U_f^2 + \frac{U_m^2 - U_f^2}{1 + (f_f/f_m)^{1/2}(2D_r)^{-2}} e^{az}\right]^{1/2}$$
$$V_m = \left[U_m^2 - (f_f/f_m)^{1/2}(2D_r)^{-2}\frac{U_m^2 - U_f^2}{1 + (f_f/f_m)^{1/2}(2D_r)^{-2}} e^{-bz}\right]^{1/2} \tag{7}$$

The above gives, analytically, the lateral variations of the vertically-averaged velocity, on the flood plain and in the main channel. It indicates that the lateral momentum exchange between the lower-speed flow on the flood plain and the higher-speed flow in the main channel leads to an increase of the velocity - and subsequently an increase of the bottom shear stress, as calculated with Eq.2 - on the flood plain, and a decease of the velocity and the bottom shear stress in the main channel. The vertically-averaged velocities can be calculated with known water depth, friction coefficient, slope and the coefficient ϑ, which in turn can be evaluated from existing laboratory data.

CONCLUSIONS

A simple momentum analysis has been presented above. It is seen from the obtained analytical solutions (Eq.7) that, due to the lateral momentum transfer from the main channel to the flood plain, the velocity - hence also the bottom shear stress - on the flood plain is increased, while the one in the main channel is decreased. This is in agreement with what has been reported in the literature.

REFERENCES

[1] Knight, D. W. and K. Shiono (1990), "Turbulence measurements in a shear layer region of a compound channel", J. Hydr. Res., Vol.28, No.2, pp.175-96.

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