II-567 Renormalization Group Analyses of Turbulence Problem

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1. Introduction

The statistical theory of turbulence is known as an approach to resolution of the turbulence problem. In 1977, the technique of renormalization group analysis discussed in this paper was introduced by Forster et al.(1) to explain advection diffusion in turbulent flow. Then, in 1986, Yakhot - Orszag(2),(3) structured a turbulent flow by renormalization group analysis, and in 1989, simulation of turbulent flow by LES model obtained from renormalization group analysis was tried by Yakhot et al.(4). However, the turbulent flow analysis obtained by Yakhot - Orszag(2),(3),(4) was not a correct one since there was an error in expansion of the equation for turbulent flow. Structuring of a turbulent flow by renormalization group analysis is done in this paper correcting the error in the theorization of Yakhot - Orszag(2),(3),(4)

2. Fundamental Equation

The Navier - Stokes equation for wave number space will be as follows :

$$v_l(\hat{k}) = G_0(\hat{k}) f_l(\hat{k}) - \frac{1}{2} i \lambda_0 G_0(\hat{k}) P_{lmn}(\mathbf{k}) \int v_m(\hat{q}) v_n(\hat{k}-\hat{q}) \frac{d\hat{q}}{2\pi^{d+1}} \quad \text{-------- (1)}$$

At this time,

$$G_0(\hat{k}) = (-i\omega + \nu_0 k^2)^{-1} \quad \text{-------- (2)}$$

$$P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j / k^2 \quad \text{-------- (3)}$$

$$P_{ijk}(\mathbf{k}) = k_k P_{ij}(\mathbf{k}) + k_j P_{ik}(\mathbf{k}) \quad \text{-------- (4)}$$

where, Gaussian distribution is considered for external force, and

$$\langle f_i(\mathbf{k},\omega) f_j(\mathbf{k}',\omega') \rangle = (2\pi)^{d+1} 2 D_0 k^{-y} P_{ij}(\mathbf{k}) \delta(\mathbf{k}+\mathbf{k}') \delta(\omega+\omega') \quad \text{-------- (5)}$$

Furthermore, $k = (\mathbf{k}, \omega)$, $\lambda_0 = 1$, and $y > -2$ are to be taken .

3. Formulation by Renormalization Group

The concept of renormalization group analysis is introduced in Eq. (1) to alter the form of the equation. Divide velocity into higher than wave number and lower than wave number. By repeatedly substituting velocity higher than wave number, and eliminating, the expanded equation by velocity lower than wave number is obtained. Cosidered the second order term by the following integration, this integration is component higher than wave number, physically, means inclusion of the influences of small eddies on large eddies in turbulent flow. Renormalization is done using scale transformation. Rearranging by substituting the scale transformation, the following result :

$$v_l'(\hat{k}') = G_r(\hat{k}')f_l'(\hat{k}') - \frac{1}{2}i\lambda(r)G_r(\hat{k}')P_{lmn}(k')\int v_m'(\hat{q}')v_n'(\hat{k}'-\hat{q}')\frac{d\hat{q}}{2\pi^{d+1}} \quad \text{-------- (6)}$$

where ,

$$G_r(\hat{k}') = [-i\omega + \nu(r)(k')^2]^{-1} \quad \text{-------- (7)}$$

$$f'(\hat{k}') = f^{<}(\hat{k})\, e^{\alpha(r)}\xi^{-1}(r) \quad \text{-------- (8)}$$

$$\lambda(r) = \lambda_0\, \xi(r)\, e^{-(d+1)r} \quad \text{-------- (9)}$$

$$\nu(r) = \nu_0\, e^{\alpha(r)-2r} \quad \text{------- (10)}$$

External force f'(k') is given by a correlation such as the following:

$$\langle f_i'(k,\omega) f_j'(k',\omega')\rangle = (2\pi)^{d+1} 2D' P_{ij}(k)\delta(k+k')\delta(\omega+\omega') \quad \text{------- (11)}$$

by which ,

$$D' = \frac{D_0 \exp[3\alpha(r) + (d+y)r]}{\xi^2} \quad \text{------- (12)}$$

is obtained. Since fluctuation D_0 is not affected even when scale transformation is done $D' = D_0$ can be used. Hence , from this , the scale parameter will be as follows :

$$\xi = \exp\left[\frac{3}{2}\alpha(r) + \frac{d+y}{2}r\right] \quad \text{------- (13)}$$

To sum up , the kolmogorov constant corrected formulation of the energy spectrum E(k) may be written as follows :

$$E(k) = C_K\, \bar{\varepsilon}^{\frac{2}{3}} k^{-\frac{5}{3}} \quad \text{------- (14)}$$

$$(C_K = 1.605) \quad \text{------- (15)}$$

This kolmogorov constant C_k is in the range (C_k = 1.3 to 2.3) which has been determined through experiments .

4. Conclusion

Structuring of turbulent flow by renormalization group analysis was done correcting the error in the theory of Yakhot - Orszag . It was thereby succeeded in theoretically determining the constants of turbulent flow as a result .

Reference

(1) Forster , D. , Nelson , D. , and Stephen , M. , phys. Rev. , A16 (1977) , 732
(2) Yakhot , V. , and Orszag , S.A. , Renormalization Group Analysis of turbulence , phys. Rev. , (1986) , 1722 - 1724
(3) Yakhot , V. , and Orszag , S.A. , Renormalization Group Analysis of turbulence , J. Sci. Comput. , 1 - 1 , (1986) , 3 - 51
(4) Yakhot , A. , and Orszag , S.A. , Yakhot , V. , and Israeli , M. , Renormalization Group Formulation of Large - Eddy Simulation , J. Sci. Comput. , 4 - 2 , (1989) , 139 - 158