II-566 A NEW FORMULATION OF THE QUICKEST SCHEME BY A TWO-STEP APPROACH

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1. Introduction

The QUICKEST scheme was developed by Leonard in 1979 for highly convective flow-specifically, for conditions in which the Peclet number is much larger than unity. It is an explicit, third-order accurate scheme. It has been shown that the QUICKEST scheme produces less dispersion than Lax-Wendroff, second-order upwinding and so on. The algorithm is based on a control-volume formulation with cell-face values of dependent variable written in terms of a quadratic interpolation. In this paper, a fresh approach to obtain the higher-order QUICKEST scheme with relative ease is presented. This alternative to producing the QUICKEST scheme is to combine a two-step predictor-corrector method with the higher-order transient interpolation modeling. It can be seen that this approach is simpler, more straight as compared with the original one. After the formulation of the two-step QUICKEST scheme, the attention is focused on the superficially simple but embarrassingly difficult problem of unsteady one-dimensional pure advection of a step discontinuity in value. To turn the two-step QUICKEST scheme into a working code, one is immediately faced with the problem of how to discretize the Eq.(1) at the grid point nearest to the upstream boundary, since the use of QUICKEST at this grid point would require values of the flow variables at a pseudonode. In this work, to avoid using pseudonode, an alternative strategy is discussed which is to use a discretization method on a three-point(or two-point) support at grid point $i = 2$. Four schemes at grid point 2 are tested and compared each other in this paper. Moreover, the universal limit is used to banish unphysical over-undershoots without corrupting the expected accuracy of the underlying method.

2. The formulation

Unsteady one-dimensional pure convection at constant velocity is described:

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} = 0 \tag{1}$$

The new approach consists of two steps. In the first step, the Lax scheme is applied to Eq.(1).

$$\phi_{i+1/2}^{n+1/2} = \phi_{i+1/2}^{n} - \frac{U\Delta t}{2\Delta x}(\phi_{i+1}^{n} - \phi_{i}^{n});\quad \phi_{i-1/2}^{n+1/2} = \phi_{i-1/2}^{n} - \frac{U\Delta t}{2\Delta x}(\phi_{i}^{n} - \phi_{i-1}^{n}) \tag{2}$$

where $\phi_{i+1/2}^{n}, \phi_{i-1/2}^{n}$ are assumed to be:

$$\phi_{i+1/2}^{n} = \frac{1}{2}(\phi_i + \phi_{i+1}) - A(\phi_{i-1} - 2\phi_i + \phi_{i+1}) \tag{3}$$

$$\phi_{i-1/2}^{n} = \frac{1}{2}(\phi_{i-1} + \phi_i) - A(\phi_{i-2} - 2\phi_{i-1} + \phi_i) \tag{4}$$

where A is an adjustable constant, and will be determined later to eliminate wiggles.
In the second step, the Leap-Frog scheme is employed:

$$\phi_i^{n+1} = \phi_i^n - \frac{U\Delta t}{\Delta x}(\phi_{i+1/2}^{n+1/2} - \phi_{i-1/2}^{n+1/2}) \tag{5}$$

Combining these two steps and using Taylor expansion, the modified partial differential equation is obtained as below:

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} = \Delta x^2[A + \frac{1}{6}(\frac{U\Delta t}{\Delta x})^2 - \frac{1}{6}]\frac{\partial^3 \phi}{\partial x^3} + o(\Delta x^2) \tag{6}$$

In order to get wiggle-free solution, A is determined:

$$A = \frac{1}{6}[1 - (\frac{U\Delta t}{\Delta x})^2] \tag{7}$$

It is now clear that this two-step algorithm is equivalent to the QUICKEST scheme in the case of pure convection flow.

3. Results

Four different schemes are used to discretize the equation at grid point 2. These are Lax-Friedrichs, First-order Upwind, Lax-Wendroff as well as a modified Leart scheme by authors. Solutions at other interior nodes are obtained from the two-step QUICKEST plus a universal limit proposed by Leonard. It should be noted that the high-resolution of the limited QUICKEST scheme would not be affected by the treatment at the point $i = 2$ if the leading-edge of a step profile is initially positioned at the grid point $i \geq 4$. If the initial value is 0 for $i \geq 2$, and 1.0 at $i = 1$, the effect of treatment at $i = 2$ on step solution occurs as shown in Fig.1a, 1b. Fig.1a, 1b corresponds to the time T=0.004 and T=0.445(after 90 time steps),respectively. A moderate Courant number C=0.5 is taken in the computations. It can be seen from the Fig.1a that the second-order discretization at $i = 2$ produces a sharper step profile as compared with first-order treatment at $i = 2$. However, slight overshoots are observed associated with second-order treatments at $i = 2$. Numerical results reveal that the use of the modified Leart scheme at $i = 2$ is identical to the use of the Lax-Wendroff scheme. Besides, the difference resulted from different treatment at $i = 2$ is found to diminish as time is advancing(see Fig.1b).

4. Conclusions

A two-step predictor-corrector algorithm for hyperbolic equation has proven equivalent to QUICKEST. Therefore, one can conclude that the QUICKEST scheme belongs to the family of two-step predictor-corrector methods. Meanwhile, discretization schemes at grid point nearest to the upstream end are discussed in connection with the pure advection problem. It is found that the second-order treatment at $i = 2$ appears to be satisfactory.

Reference

[1] B.P.Leonard,"A Stable and Accurate Convective Modelling Procedure based on Quadratic Upstream Interpolation," Comput. Methods Appl. Mech. Eng., 19, p.59, 1979.

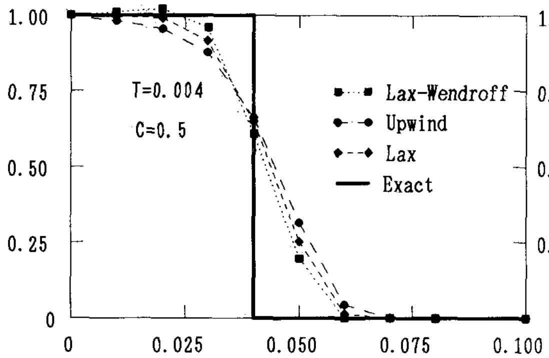

Fig.1a

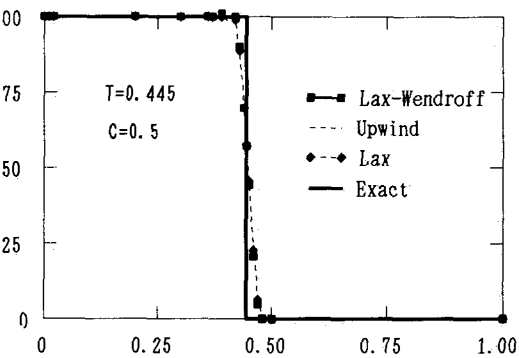

Fig.1b