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Comparison of Discretization Schemes of an Advection-Diffusion Equation

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1. Introduction

The movement of contaminant in flowing water is governed by "Advection-Diffusion Equation" and a highly accurate numerical scheme which is not suffered from numerical diffusion, is desired to solve it. Among various numerical methods available, three discretization schemes were chosen to investigate their performance in solving the advection-diffusion equation. They are (1) Power Law Differencing Scheme (PLDS) [1], (2) Quadratic Upstream Interpolation for Convective Kinematics (QUICK) [2], and (3) QUICK with Estimated Streaming Terms (QUICKEST) [2]. Each of these schemes uses control volume formulation. One case of advection-diffusion was numerically solved and the results are compared with an exact analytical solution.

2. Basic Equation

Unsteady one dimensional advection-diffusion equation of a scalar ϕ is described by,

$$\frac{\partial \phi}{\partial t} + \frac{\partial (u\phi)}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial \phi}{\partial x}\right) + s \qquad (1)$$

where u(x,t) is the advecting velocity, D(x,t) is the diffusion coefficient and s(ϕ,x,t) is a source term.

3. Universal Limiter

Certain restrictions are applied to control volume face value (ϕ_f) depending on local behavior of ϕ [3] which are termed as universal limiter constraints. Fig. 1(a) shows locally monotonic behavior of ϕ near a control volume face where the central, upstream and downstream node values are denoted by ϕ_C, ϕ_U and ϕ_D respectively. Fig. 1(b) depicts the same behavior but in terms of locally normalized variable $\tilde{\phi}$, which is defined as,

$$\tilde{\phi} = \frac{\phi - \phi_U}{\phi_D - \phi_U} \qquad (2)$$

Universal limiter constraints in terms of normalized variable are shown in Fig. 2.

4. Test case

The case considered for the solution of Eq. (1) is boundary value problem specified by,

$$c(0,t) = c_0, \qquad 0 < t < \infty\,; \quad c(x,0) = 0, \qquad 0 < x < \infty \qquad (3)$$

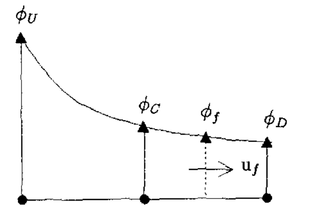
The exact solution is given by

$$c(x,t) = \frac{c_0}{2}\left[erfc\left(\frac{x-ut}{2\sqrt{Dt}}\right) + erfc\left(\frac{x+ut}{2\sqrt{Dt}}\right) exp\left(\frac{ux}{D}\right)\right]. \qquad (4)$$

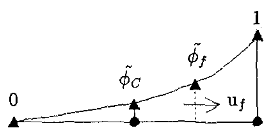
5. Results and Discussion

All the calculation were performed with Peclet number 10. The results of PLDS, unlimited QUICK and QUICKEST schemes are shown in Fig. 3. PLDS always shows large amount of numerical diffusion which increases with time and Courant number (γ) and the results are grossly inaccurate though there is no wiggle. QUICK and QUICKEST schemes are free from numerical diffusion. QUICKEST scheme gives highly accurate results but small unphysical oscillations are produced near the region of sharp gradient change although they die out with time as the sharp discontinuity is diffused. The accuracy of QUICKEST scheme is better with higher Courant number. QUICK produces generally less accurate results than QUICKEST and the wiggles are relatively much greater which do not disappear with time. Its accuracy increases with lower Courant number.

Universal limiter constraints have been applied to overcome the problem of wiggles. Fig. 4 shows the results of ULTIMATE (using universal limiter) QUICK and QUICKEST schemes. Application of universal limiter successfully alleviates the overshoots and undershoots but a new problem arises. ULTIMATE QUICKEST scheme shows loss in accuracy when the Courant number is high.



a) Original unnormalized variables.



b) Normalized variables.

Fig. 1. Locally monotonic behavior across a control volume cell.

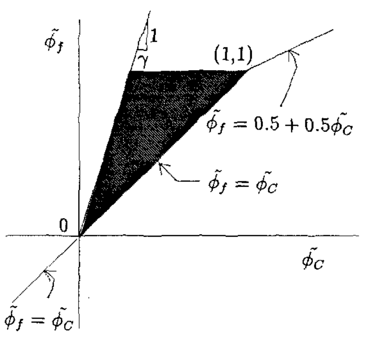


Fig. 2. Universal limiter constraints in the normalized variable diagram.

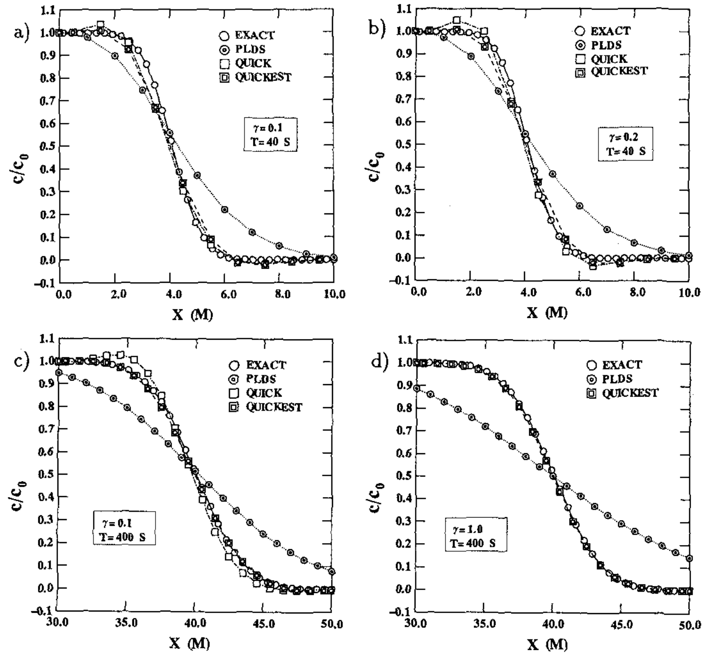


Fig. 3. Results of PLDS, unlimited QUICK and QUICKEST schemes.

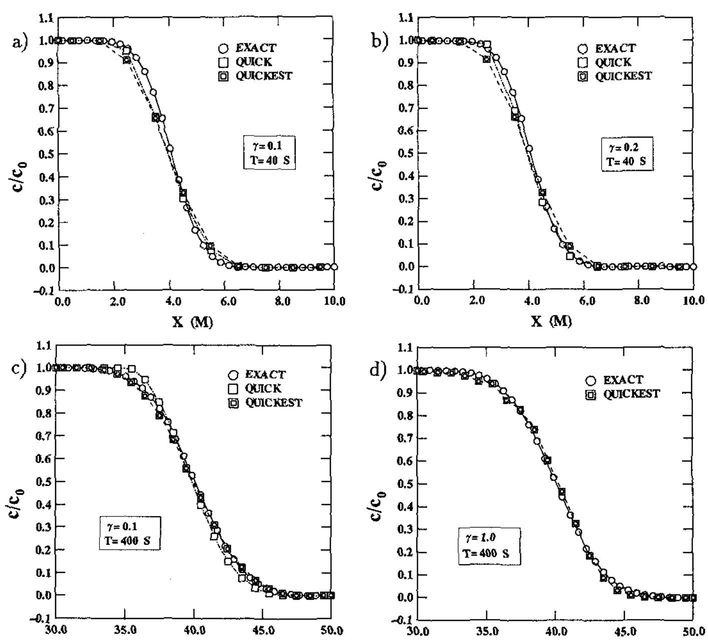


Fig. 4. Results of ULTIMATE QUICK and QUICKEST schemes.

6. References

[1] Patankar, S.V.(1980): Numer. Heat Trans. and Fluid Flow. Hemi. Publ. Corp., McGraw Hill.
[2] Leonard, B.P.(1979): Comput. Meths. Appl. Mech. Engrg., Vol. 19, No.1, pp. 59-98.
[3] Leonard, B.P.(1988): NASA TM 100916(ICOMP-88-11), 116 p.