

II-564

三段階有限要素法による非定常非圧粘性流れ解析

中央大学 ○ 江 春波(正員), 畑中 勝守(学生員)
川原 陸人(正員), 檜山 和男(正員)

1. Introduction: Based on the Taylor expansion theory, a three-step finite element method has been proposed by the author's group^[1]. This method remains the advantages of the Taylor-Galerkin method, such as the good accuracy and the uniform CFL condition^[1]. In contrast with the Taylor-Galerkin method, the three-step finite element method does not contain any new higher-order spatial derivatives, which can be applied to solve the multi-dimensional flow problems with ease.

2. Mathematical Model: The three-step finite element method is applied to solve the incompressible flows. The time discretized formulations are

$$\begin{aligned}\frac{u_i^{n+1/3} - u_i^n}{\frac{\Delta t}{3}} &= -u_j^n u_{i,j}^n - p_{,i}^n / \rho + \nu(u_{i,j}^n + u_{j,i}^n)_{,j} + f_i^n \\ \frac{u_i^{n+1/2} - u_i^{n+1/3}}{\frac{\Delta t}{2}} &= -u_j^{n+1/3} u_{i,j}^{n+1/3} - p_{,i}^{n+1/3} / \rho + \nu(u_{i,j}^{n+1/3} + u_{j,i}^{n+1/3})_{,j} + f_i^{n+1/3} \\ \frac{u_i^{n+1} - u_i^{n+1/2}}{\Delta t} &= -u_j^{n+1/2} u_{i,j}^{n+1/2} - p_{,i}^{n+1/2} / \rho + \nu(u_{i,j}^{n+1/2} + u_{j,i}^{n+1/2})_{,j} + f_i^{n+1/2}\end{aligned}\quad (1)$$

The spatial discretizations of equations (1) can be performed by using the standard Galerkin method. By taking the divergence on both sides of the last formulations of equation (1) and introducing the incompressible constraint $u_{i,i}^{n+1} = 0$, the pressure can be solved from a derived Poisson's equation

3. The Unsteady Density Flow: A fluid of density $\rho_1 = 1.0$ occupies the left-hand two-thirds of the tank and a heavier fluid, with density $\rho_2 = 1.2$, occupies the right-hand one-third of the tank. The flow is started at $t=0$ due to the density difference. The positions of the fluids are described by the markers. The distribution of the markers at $t=6$ sec. is shown in Fig.1. The front position versus time is shown in Fig.2. The present calculation is in agreement with the experiment^[2], the accuracy is better than the Lumping finite element method^[3].

4. The Flow Past A Flat Plate : The calculated time history of the stagnation point, as shown in Fig.3, is compared with the experimental data^[4] and the other numerical results^[5]. The present calculations are in excellent agreement with those reported in the reference results.

5. Conclusions: The three-step finite element method has been extended to solve the unsteady incompressible flows. It has third-order accuracy and uniform CFL property. The present method can be efficiently applied to solve the convection dominated incompressible flows. The obtained numerical results are in good agreement with the literatures.

6. References: [1]. 江 春波 他, 日科技連第5回計算力学シンポジウム報文集, 211-218, 1991年10月, 東京. [2]. B.J.Daly et al., the physics of fluids, Vol.11 No.1, 15-30, 1968. [3]. 大宮 清隆 他, 日科技連第6回流れの有限要素解析シンポジウム, 97-94, 1985年8月. [4]. S.Taneda and H.Honji, J. Physical Society of Japan, **30**, No.1, 262-272 (1971). [5]. Y.Yoshida and T.Nomura, Int. J. for Numerical Methods in Fluids, **5**, 873-890 (1984).

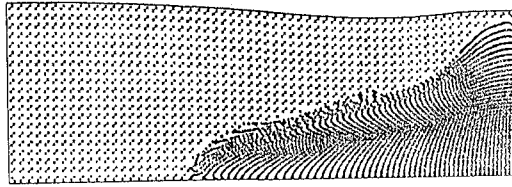


Fig.1 The density flow, $t=6.0$ sec.

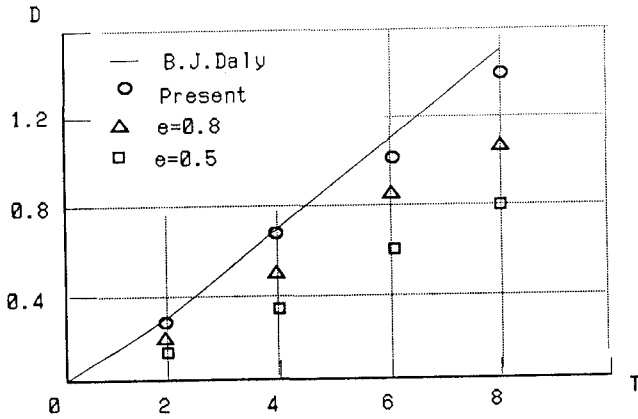


Fig.2 The front versus time (density flow)

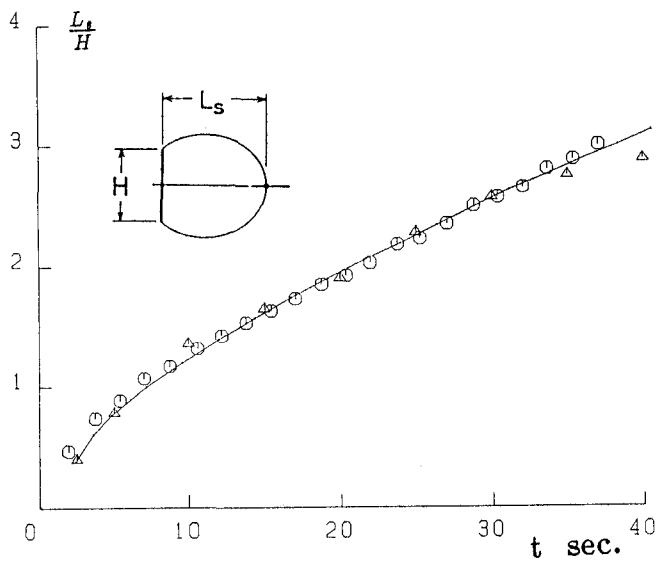


Fig.3 The time history of the stagnation point