II-563 FINITE ELEMENT NUMERICAL SIMULATION OF DENSITY FLOW

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Density flow can be described as the flow which results from the small density differences of materials. The study of density flow attracted much attention because of its wide spectrum of applications in all engineering fields. Many numerical methods have been proposed for density flow problems. Marker and cell method is most commonly used one to simulate the density induced flow problems. Daly [1], Daly and Pracht [2] have studied the density current surges for different density ratios using marker and cell method. They compared their result with experimental observations and the agreement is good. Hirt et al [3] simulated numerically the intense explosion in the atmosphere using marker particles by arbitrary Lagrangian and Eulerian method. Extending this finite difference analysis principle, Kawahara and Ohmyia [4-5], and Ramaswamy [6] have proposed finite element simulation for free surface density flow. The method developed by Kawahara and Ohmyia [4-5] is based on velocity correction principle originated by Chorin. By discretizing the equation of motion, intermediate velocity is obtained. This intermediate velocity may not satisfy the continuity equation. To correct this intermediate velocity, correction potential is introduced. One of the necessary condition for this analysis is, the normal pressure gradient at (n+1)th time should coincide with n th time value. It is difficult to keep this condition valid for all situations. In Ramaswamy's numerical simulation [6], this difficulty has been avoided by deriving pressure Poisson equation directly from the momentum equations. However, boundary condition complexities for Poisson equation still exists. Recently Hayashi et al [7] have analyzed the pressure boundary conditions for both the consistent and the directly derived pressure Poisson equations and suggested a simple way to derive Neumann pressure boundary conditions for solving the pressure Poisson equation.

Following Hayashi et al's analysis [7], we have presented a numerical method for the free surface density flow. In the first step of the present method, the pressure field is computed from the derived pressure Poisson equation. The second step is the calculation of velocity vector field from the momentum equations explicitly using the known pressure. To track the free surface position, rather than using the free surface kinematic relation followed by references [4-6], we have employed Lagrangian method. By this process, free surface position has been computed without any additional equation. This modified approach's (Eulerian mesh with Lagrangian free surface line) applicability has been demonstrated through numerical examples.

Formulation of the problem:

Using the standard summation and indicial notation, the following Navier-Stokes equations for the two different density fluids (Boussinesq approximation is assumed) can be written as

$$\frac{\partial u_i}{\partial t} + u_j u_{i,j} = -\frac{p_{,i}}{\rho_0} + \nu(u_{i,j} + u_{j,i})_{,j} + \frac{\rho}{\rho_0} f_i \tag{1}$$

$$u_{i,i} = 0 \tag{2}$$

where u_i and p are fluid velocity of coordinate x_i and pressure. The notations ρ, ρ_0, ν and f_i mean density, reference density, kinematic viscosity and body force respectively.

Initial and boundary conditions:

The surface Γ consists of two types of boundaries, that is, solid boundary Γ_1 and free surface boundary Γ_2. The initial and boundary conditions are

$$u_i(x_i, 0) = \bar{u}_i^0(x_i) \tag{3}$$

$$u_i = \bar{u}_i, \quad on \ \ \Gamma_1 \tag{4}$$

$$(-\frac{1}{\rho_0} p\delta_{ij} + \nu(u_{i,j} + u_{j,i}))n_j = \bar{t}_i, \quad on \ \ \Gamma_2 \tag{5}$$

Where n_j is unit vector normal to the boundary and bar means prescribed value.

Marker movement and Density distribution:

In the present method, marker particles directly involves in the computation at every time step. Initially marker particles have been classified according the type of fluid each represents and then each retain their classification. It is convenient to identify particle position at any time during the computation. The time advancement of particle position is done according to the following relation

$$x_i^{n+1} = x_i^n + \Delta t u_i^n \tag{6}$$

Where x_i^n is the position of marker particle at time n. u_i^n is the local velocity, it has been calculated by using the linear interpolation functions of each element. Once we know the complete information about each mesh cell, density can be calculated from these data using the following formula

$$\rho^{(i)} = \frac{n_1\rho_1 + n_2\rho_2}{n_1 + n_2} \tag{7}$$

Where $\rho^{(i)}$ represents the density of element (i), ρ_1 and ρ_2 are the densities of the two materials, and n_1 and n_2 are the numbers of particles of materials 1 and 2 in each element.
The reference density is computed from the equation

$$\rho_0 = \frac{\sum_{i=1}^{M} \rho^{(i)} A^{(i)}}{\sum_{i=1}^{M} A^{(i)}} \tag{8}$$

Where $A^{(i)}$ is an area of the finite element and M is number of elements in the computational domain.
Numerical examples:

Two test examples, mixing of two different density fluids and high density flume slide over an inclined plane filled with low density fluid have been chosen. The computation starts with zero velocity field. The computed density distribution shown by marker particles at different time points is explained in Fig.1. The figure configuration clearly shows the different density fluids mixing behaviour (heavier fluid sinks under the lighter fluid as time increases). This computation uses lumping parameter (0.9) to reduce the existing numerical damping near the interface. The shown result's close confirmation with other theoretical and experimental observations indicate presently developed algorithm's effectiveness in handling the free surface density flows.

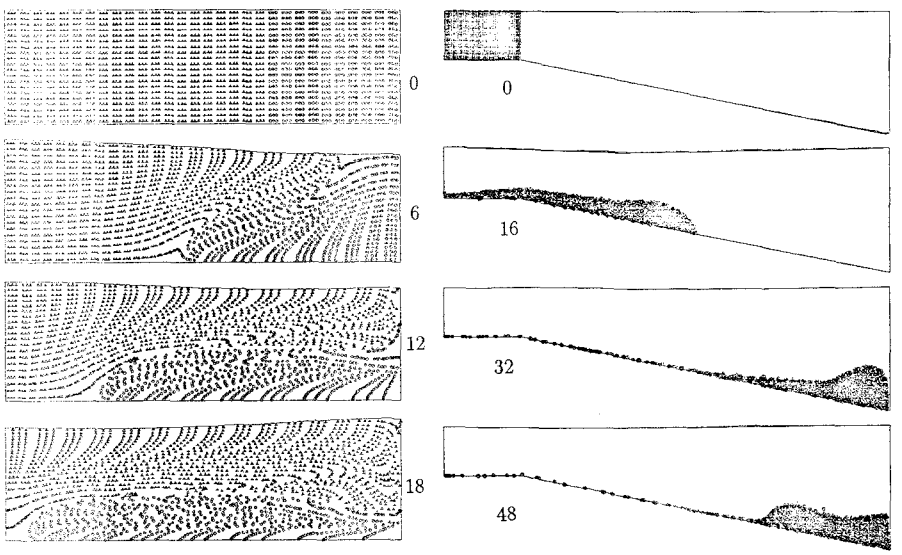

Fig.1. Density distribution shown by marker particles.

References:
1. B.J.Daly, Numerical study of two fluid Rayleigh Taylor instability, Phys. Fluids, 10, 297-307, 1967.
2. B.J.Daly and W.E.Pracht, Numerical study of density current surges, Phys. Fluids, 11, 15-30, 1968.
3. C.W.Hirt, A.A.Amsden and J.L.Cook, An Arbitrary Lagrangian Eulerian computing method for all flow speeds, J. Comput. Phys., 14, 227-253, 1974.
4. M.Kawahara and K.Ohmiya, A finite element method for density flow using velocity correction method, Theo. Appl. Mech., 34, 43-47, 1984.
5. M.Kawahara and K.Ohmiya, Finite element analysis using the velocity correction method, Int. j. num. methods fluids,5,981,1985.
6. B.Ramaswamy, Numerical simulation of unsteady viscous free surface flow, J. Comput. Phys., 90, 396-430, 1990.
7. M.Hayashi, K.Hatanaka and M.Kawahara, Lagrangian finite element method for free surface Navier Stokes flow using fractional step principle, Int.j.num.meth.fluids,12, 805-840, 1991.