II-561 NUMERICAL COMPUTATIONS BY A NONLINEAR WAVE MODEL

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1 Introduction

The Boussinesq-type equations governing the propagation of arbitrary, long-wave disturbances over a slowly varying bathymetry by small to moderate amplitude waves have been derived by Peregrine in 1967. Since then, various numerical models based on these equations have been developed (Abbott et al., 1978; Schaper and Zielke, 1984; Madsen and Warren, 1984; Larsen et al., 1984; Berenguer et al., 1986; Yu et al., 1987; Smallman and Cooper, 1989; Maa, 1990; Madsen et al., 1991). The different forms of the Boussinesq-type wave equations and their applicability is discussed in McCowan (1981). Abbott et al. (1984) investigated the accuracy of the terms in some of these forms. The attractiveness of the Boussinesq equations for numerical modelling is due to the fact that the Boussinesq theory can be considered as the most uniformly valid theory for finite amplitude non-breaking water waves in shallow water.The present paper describes some of the initial results of a model based on Boussinesq-type water wave equations which includes nonlinear two-dimensional wave computations.

2 Governing Equations

For incompressible irrotational flow in two horizontal direction, the classical form of the Boussinesq continuity and momentum equations used by Abbott et al. (1984) are

$$\frac{\partial \eta}{\partial t}+\frac{\partial p}{\partial x}+\frac{\partial q}{\partial y}=0 \tag{1}$$

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}(\frac{p^2}{d})+\frac{\partial}{\partial y}(\frac{pq}{d})+gd\frac{\partial \eta}{\partial x}=\frac{1}{3}h^2(\frac{\partial^3 p}{\partial x^2\partial t}+\frac{\partial^3 q}{\partial x\partial y\partial t}) \tag{2}$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}(\frac{pq}{d})+\frac{\partial}{\partial y}(\frac{q^2}{d})+gd\frac{\partial \eta}{\partial y}=\frac{1}{3}h^2(\frac{\partial^3 p}{\partial x\partial y\partial t}+\frac{\partial^3 q}{\partial y^2\partial t}) \tag{3}$$

where d is the total water depth, h is the still water depth, $\eta=d-h$, and p and q are the depth-integrated velocity components in the x- and y-direction respectively. The wave dispersion terms on the right side of Eqs. (2) and (3) are applicable for a constant depth or for a mild slope with discretely constant depth.

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3 Numerical Computations

A finite difference ADI scheme was used to solve the governing equations over a region which, for simplicity, has been made to have a uniform grid space Δs in both horizontal directions. The scheme can be easily extended for a variable grid region. A time step Δt is divided into two: from $t{=}n$ to $t{=}n+\frac{1}{2}$ and then from $t{=}n+\frac{1}{2}$ to $t{=}n+1$. The form of the finite difference equations used and the numerical procedure are found in Maa (1990). A time series $\eta(t)$ is used to generate the waves propagating in one direction from an incident boundary. Wave reflection is described by letting $p{=}0$ or $q{=}0$ at points where the fully reflecting boundary is located. An absorbing boundary is described by letting

$$p^t(x,y)=p^{t-\tau}(x-\Delta s,y) \tag{4}$$

or

$$q^t(x,y)=q^{t-\tau}(x,y-\Delta s) \tag{5}$$

where $x-\Delta s$ or $y-\Delta s$ is the adjacent inner point from point x or point y, $\tau{=}\Delta s/c$, and c is the wave celerity.

4 Discussion of Model Results

The model was applied to 1-D linear and nonlinear wave simulation on a region of constant depth (Case 1), and on the same region with a constant slope (Case 2). Finally, 2-D linear and nonlinear wave simulations were done on a region of constant depth (Case 3).

In Case 1, the region is a 5 × 70 rectangular grid with water depth h and wave period T so chosen such that $kh<1$. The incident waves were coming from the eastern boundary and the northern and southern boundaries were kept to be fully reflecting boundaries. Computations were made for full reflection and then full absorbtion at the western boundary. The condition for Case 2 is similar to Case 1 except that instead of a constant depth, a minimum depth for which $kh<1$ and a slope up to the incident boundary was used. Beach slopes s used varied from 0.02 to 0.1. For Case 3, a 40 × 70 rectangular grid with a constant depth and wave period for which $kh<1$ were used. Both the northern and western boundaries were fully absorbing boundaries while the southern boundary was kept to be a fully reflecting boundary. Incident waves were made to come from the eastern boundary. A fully reflecting breakwater 45 degrees oblique with the boundaries was placed inside the region. In all cases, the incident boundary was made to allow the absorbtion of the reflected wave components. The waves are monochromatic and non-breaking. The grid spacing Δs was set to

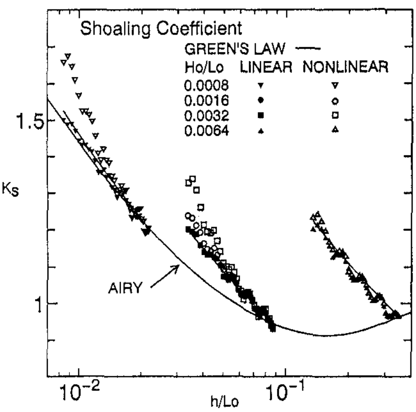

Figure 1: Case 2 Shoaling coefficient K_S for various H_o/L_o

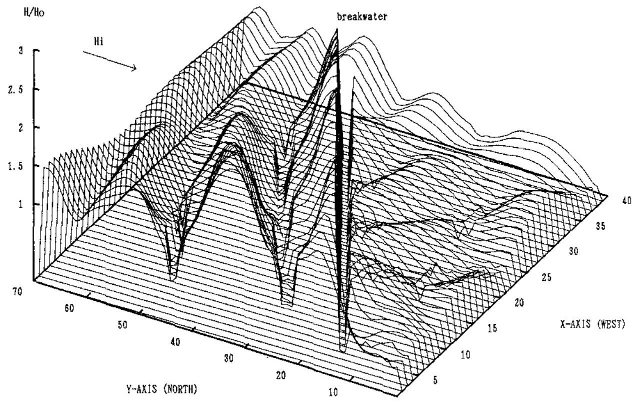

Figure 2: Case 3 Surface envelope of H/H_o from nonlinear wave computations with T=20 s, d=30 m, H_i=1 m, Δs=15 m, Δt=0.8 s

1/20 of the wavelength L and Δt was 1/25 of the wave period T.

The results of computations for Case 1 indicate good agreement with the expected results. With a fully reflecting western boundary, standing waves were observed in the region; while with a fully absorbing boundary, progressive waves were observed. Figure 1 shows the shoaling coefficient K_S from Case 2 computations for different values of H_o/L_o. For each H_o/L_o value, linear wave modelling results adhered closely to Green's Law. The nonlinear wave modelling gave *higher* K_S values. The base of each line representing Green's Law at the incident boundary can be seen to follow Airy's shoaling coefficient.

Figure 2 shows the surface envelope of H/H_o for the whole region for Case 3. The simulations gave reasonable results particularly around the vicinity of the breakwater. Waves in front of the breakwater were observed to propagate in the parallel direction and were standing waves in the perpendicular direction from the breakwater. Nonlinear computations gave higher wave heights when compared with linear computaions.

5 Conclusions

A wave model capable of simulating wave in 1-D and 2-D is presented. The absorbing boundary condition was described by using the history of the depth-integrated velocity components of adjacent points. Wave nonlinearities which are included in the convective and gravity terms of the Boussinesq equations have been shown to be related to h/L_o and H_o/L_o. The model can give reasonable results on linear and nonlinear wave propagation and diffraction. Nonlinear computation including the dispersion terms of the Boussinesq equations will be presented at the conference.

6 References

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