II-525 Wave Diffraction due to a Rubble Trench

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1. Introduction

Rubble structures, such as rubble mound breakwaters, are highly dissipative. Therefore, rubble in the trench may also result in high energy dissipation. A boundary element model is developed for the interaction of waves with an infinite length trench which can contain rubble. In this paper, the effect of rubble in the trench on wave transmission are examined. Permeability is estimated as a function of both porosity and stone size. Energy dissipation due to rubble in the trench is estimated by using linearized friction coefficient.

2. Governing equations

The fluid domain is divided into 4 regions as shown in Fig.1. The trench has dimensions (d-h)×b in x-z plain and is infinitely long in y-direction. The water depth outside the trench is uniform, h. The incident wave has an angle θ_1 to the positive x-axis and propagates from left to right. The fluid is assumed to be incompressible and irrotational.

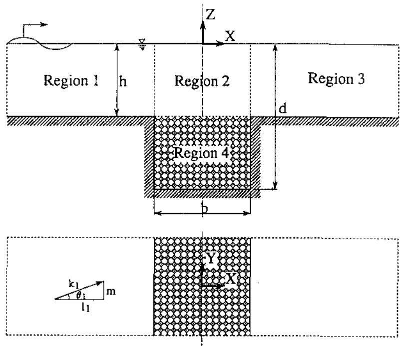

Fig.1 Definition Sketch

2.1 Equation of fluid motion in the porous domain

The motion of the fluid in a rubble is described in terms of the seepage velocity and local pressure as Eq.(1), where,

$\vec{v}$: local seepage velocity vector , p : local pressure

The resistance forces are primarily due to drag and inertia. Previous studies have shown the drag forces are modeled by the Forchheimer equation.

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla(p+\rho g z) + (\text{resistance forces}) \qquad (1)$$

$$[\text{drag force}] = \alpha\vec{v} + \beta|\vec{v}|\vec{v} \qquad (2)$$

The drag forces were modeled by Sollitt and Cross[3] as Eq.(3), where,

υ : kinematic viscosity , K : intrinsic permeability

Cf :turbulent resistance coefficient , ε : porosity

$$\alpha\vec{v} + \beta|\vec{v}|\vec{v} = \frac{\nu\epsilon}{K}\vec{v} + \frac{C_f\epsilon^2}{\sqrt{K}}|\vec{v}|\vec{v} \qquad (3)$$

The terms representing the drag forces in the equation of motion are linearized based on Lorent'z hypothesis of equivalent work. The terms responsible for drag forces are replaced by a linear term in the velocity as Eq.(4). where,

$$S = 1 + \frac{1-\varepsilon}{\varepsilon}C_M$$

$$S\frac{\partial\vec{v}}{\partial t} = -\frac{1}{\rho}\nabla(p+\rho g z) - f\omega\vec{v} \qquad (4)$$

If $\vec{v}$ is taken as simple harmonic, **f** is expressed by Eq.(5).

where, $\vec{q}$: average velocity in the rubble

Since $\vec{q}$ is unknown, iteration is required to determine **f**.

$$f = \frac{1}{\omega}\left[\frac{\nu\epsilon}{K} + \frac{8}{3}\frac{C_f\epsilon^2}{\sqrt{K}}\frac{|\vec{q}|}{\pi}\right] \qquad (5)$$

2.2. Boundary value problem

The velocity potential Φ (x,y,z) must satisfy the 3-D Laplace's equation. Snell's law for refraction due to change in the water depth yields, m=const. for each region. Therefore, the governing equation for ϕ (x,z) are the modified Helmholtz equation.

$$\frac{\partial^2\phi_j}{\partial x^2} + \frac{\partial^2\phi_j}{\partial z^2} - m^2\phi_j = 0 \qquad (6)$$

The matching conditions between the porous trench and upper fluid region may be summarized as Eq.(7).

$$\frac{\partial\phi_2}{\partial z} = \epsilon\frac{\partial\phi_4}{\partial z}$$
$$\phi_2 = (S+if)\phi_4 \quad , \quad i=\sqrt{-1} \qquad (7)$$

3. Permeability

The dissipation of energy in the rubble is strongly dependent on the permeability. The permeability itself depends on the grain size and porosity. Here, simple relationships among these are obtained. The engineering permeability **k** is related to the intrinsic permeability **K** as,

$$v = K\frac{\rho g}{\mu}\frac{dh}{dx} = k\frac{dh}{dx} \qquad (8)$$

According to data from the previous studies, a relationship between k and stone size D is obtained. This relationship is converted to one between K and D as shown in Fig.2 A best fit can be obtained by a least square analysis on the logarithm of the values. where, it is assumed that

$$\mu = 1.14\times 10^{-3}\mathrm{Ns/m^2},\ \rho = 1026\mathrm{kg/m^3} \quad \text{for } t=15^\circ$$

Permeability can be expressed as a function of the porosity ε as

$$K(\varepsilon) = C\,\frac{\varepsilon^3}{(1-\varepsilon)^2} \qquad (9)$$

C can be determined by using a known value of the permeability for known porosity and stone size. Finally, the permeability can be expressed as a function of both porosity and stone size.

$$K(\varepsilon,D) = 4.422\times 10^{-9}\,D^{1.57}\,\frac{\varepsilon^3}{(1-\varepsilon)^2} \qquad (10)$$

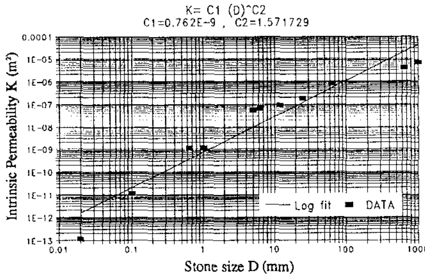

Fig.2 Relationship between K & D

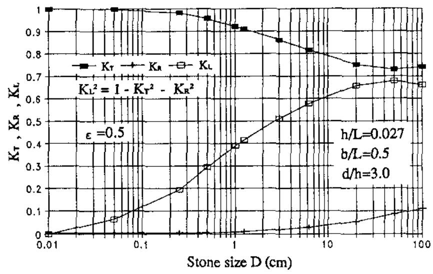

Fig.3 Dependency of K_T, K_R and K_L on the stone size

4. Results and Discussions

Fig.3 shows variations of K_T (transmission), K_R (reflection) and K_L (loss) with the stone size. As expected, when the stone size is very small, the trench effect is quite small. That is, near perfect transmission occurs. However, as the stone size increases, energy dissipation effects occur.

The velocity inside the rubble is not proportional to the wave height. As shown in Fig.4, K_T, K_R and K_L are not linear in the wave height. Fig.5 shows K_T as a function of the incident wave angle for the open trench and the rubble trench. As can be seen, K_T for the rubble trench are nearly constant up to 40°. There is no special angle which wave transmits perfectly as the case of open trench.

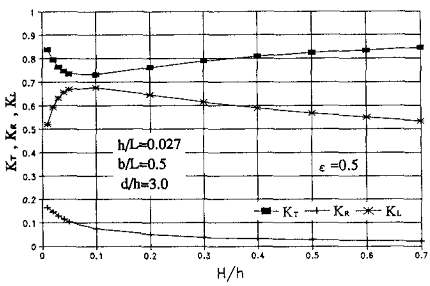

Fig.4 Dependency of K_T, K_R and K_L on the wave height

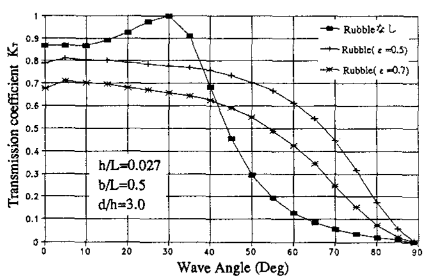

Fig.5 Dependency of K_T on the wave angle

5. Conclusions

(1) Energy dissipation due to rubble in the trench is evaluated by using a linearized friction coefficient for a variety of stone sizes and porosity of the rubble.

(2) The intrinsic permeability is derived as a function of porosity and stone size.

(3) Although the response of oblique incident waves over the open trench are sensitive to the angle, the response of the rubble trench is much less sensitive to the wave angle.

References

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3) Sollitt, C.K. and Cross, R.H. 1972,"Wave transmission through permeable breakwaters", *Proceedings of the 13th Coastal Engineering Conference, ASCE*,pp.1827-1846