I-628 Time delay and its compensation in active control of structures

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1. INTRODUCTION : Time delay occurs mainly because of on-line data acquisition, filtering, control force calculation, and the actuator time delay. The effect of time delay on the stability of a SDOF system with time delayed feedback has been presented earlier by the authors by deriving an explicit formula for maximum allowable time delay (when the structure becomes unstable)[1]. It was shown that the maximum allowable time delay (β_{max}) depends on natural period (T_0), structural damping and feedback gains. For direct velocity feedback control, for an undamped system, it was shown that $\beta_{max} / T_0 = 0.25/(\sqrt{\zeta_c^2+1}+\zeta_c)$, ζ_c is the active damping ratio. We observe from this equation that β_{max} decreases as we increase the active damping or control higher modes with low time periods. In this paper, we present a mehtod to compensate time delay by modelling it as transportation lag. We demonstrate different aspects of the method by a numerical example.

2. TIME DELAY COMPENSATION: The equation of motion for the n dof controlled structure with time delay β in the control signal can be written in the state space form as,

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y}(t) + \mathbf{B}\mathbf{u}(t-\beta) + \mathbf{E}\mathbf{f}(t) \qquad (1)$$

Where $\mathbf{A}$ and $\mathbf{B}$ are standard state space matrices, $\mathbf{u}(t)$ is r control force vector (r is the number of controllers), $\mathbf{E}$ is external force location matrix and $\mathbf{f}(t)$ is external excitation vector.

Referring to Fig.1, time delay in control signal can been modelled as transportation lag, as proposed by Hiratsuka et al.[2], during the flow of signal from the computer through the actuator to the structure. Denoting a control signal flow parameter $\mathbf{q}(v,t)$ and length along the unit length pipeline as z, we can write the continuity equation for the flow as in Eq.(2).

$$\frac{\partial \mathbf{q}(z,t)}{\partial t} + \frac{1}{\beta}\frac{\partial \mathbf{q}(z,t)}{\partial z} = 0, \; z \in [0,1] \qquad (2)$$

where, $\mathbf{q}(0,t) = \mathbf{u}(t)$, $\mathbf{q}(1,t) \equiv \mathbf{v}(t) = \mathbf{u}(t-\beta)$ (3)

and $1/\beta$ in the above equation is the velocity of control signal flow.

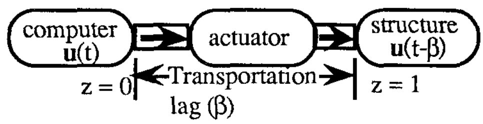

Fig. 1 Time delay modelled as transportation lag.

Using finite difference approximations, we can write the continuity equation in discretized form as,

$$\frac{\partial(\mathbf{q}(z_j,t) - \mathbf{q}(z_{j-1},t))}{\partial t} + \frac{m}{\beta}\frac{(\mathbf{q}(z_j,t) - \mathbf{q}(z_{j-1},t))}{\partial z} = 0, \; j = 1.2.....m \qquad (4)$$

Here, m is the number of discretizations.

Now, we combine Eq.(1),(3) and (4) and and write the augmented system as,

$$\dot{\hat{\mathbf{y}}}(t) = \hat{\mathbf{A}}\hat{\mathbf{y}}(t) + \hat{\mathbf{B}}\mathbf{u}(t) + \hat{\mathbf{E}}\hat{\mathbf{f}}(t) \qquad (5)$$

$$\text{where, } \hat{\mathbf{y}}(t) = [\; \mathbf{y}(t) \;\; \mathbf{q}_1(t) \;\; . \;\; . \;\; \mathbf{q}_j(t) \;\; . \;\; . \;\; \mathbf{v}(t) \;]^T \qquad (6)$$

is 2n+(r x m) augmented state vector. For detailed description of the modelling and augmented matrices/vectors, please refer to the work by Agrawal et al.[1].

By choosing a proper response penalty matrix $\mathbf{Q}$ and control cost penalty matrix $\mathbf{R}$, we do the linear optimal control of the system represented by Eq.(5) using quadratic performance index and obtain the control law as,

$$\mathbf{u}(t) = G_1 \mathbf{x}(t) + \sum_{j=1}^{m} G_{2,j}\mathbf{u}(t - \frac{j\beta}{m}) \qquad (7)$$

where, G_1 and $G_{2,j}$ are gain matrices obtained by solving the algebraic Riccati matrix equation[3]. We observe from this equation that the present control law utilizes both the response of the structure as well as past control force inputs whereas the optimal control without time delay utilizes only the response of the structure. The stability of the time delay compensated system is guaranteed if the original system defined by Eq.(1) is controllable and observable[1].

3. NUMERICAL EXAMPLE: We take an example of SDOF system with 1.0 sec undamped natural period, 2.0 % structural damping and 0.3 sec time delay and demonstrate the time delay compensation, response reduction and control energy requirements using a single actuator. Weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are chosen such that $\mathbf{R} = 0.1$ and $\mathbf{Q}_{11} = 3.4$. Other elements of $\mathbf{Q}$ matrix are zero.

Fig. 2 shows response histories of the system subjected to harmonic loading. If the time delay is not compensated, the system becomes unstable. Compensating time delay by the present technique, the system is stable and vibration control is significant (83.50 %). Although, the response reduction is 92.75 % when there is no time delay.

However, the control is not effective in the beginning. we observe this from the response of the system subjected to 43.4 kN impulsive force, shown in Fig. 3. The reason may be that complete past control force information is not available during the first β seconds of control application.

Fig.4 shows the response time history of the system subjected to El Centro N-S 1940 earthquake. We see that the method works well for random seismic loads as well. We also confirmed that the uncompensated system became unstable when the time delay exceeded β_{max}.

In the present technique, time-delay has been discretized into *m* segments. The response reduction and increase in required control energy calculated for various values of *m*, under harmonic loading, are shown in Fig.5. We observe that response reduction increases and the required control energy decreases with increase in *m*. This is because of reduced approximations involved in writing equation (4). However, we obtain less response reduction in time delay compensated case compared to the no time delay case, while the required control energy approaches the same value in the two cases. It was found by trial and error that by using $\mathbf{R} = 10^{-6}$ and $\mathbf{Q}_{11} = 7.3$ for time delay compensation case, the response reduction obtained is the same as with zero time delay case with weighting mentioned in the beginning of the section. But, this results in 19 % increase in control energy.

The increase in control energy for different time delays (β/β_{max}) has been shown in Fig.6. The increase has been calculated such that the response of the time delay compensated system is same as the response of the system for zero time delay and weightings $\mathbf{Q}_{11} = 3.4$ and $\mathbf{R} = 0.1$. The weightings for the compensated system had to be changed and were chosen iteratively. We observe not much increase in the control energy with increase in time delay.

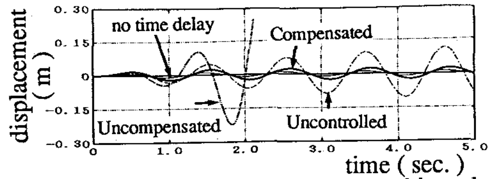

Fig. 2 Response of the system subjected to harmonic loading.

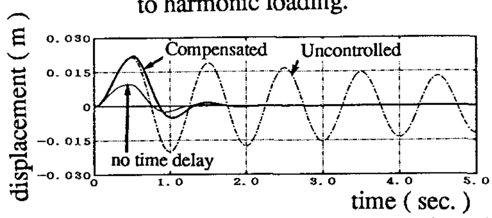

Fig. 3 Response of the system subjected to impulsive loading.

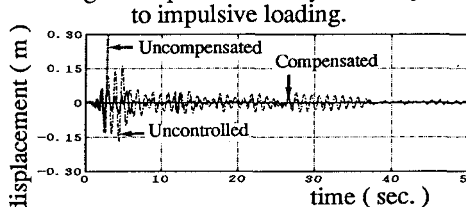

Fig. 4 Response of the system subjected to ElCentro N-S 1940 loading.

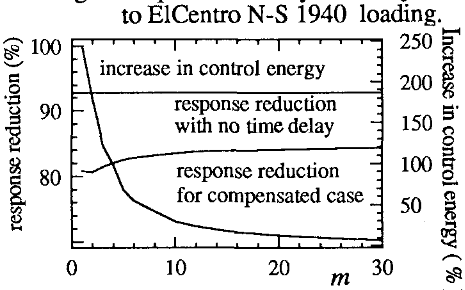

Fig. 5 Response reduction and increase in control energy versus *m*.

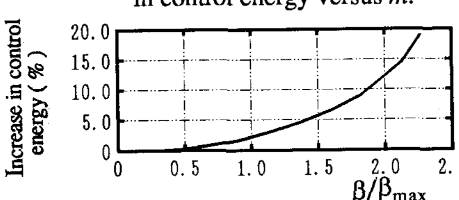

Fig. 6 Increase in control energy with time delay.

4. CONCLUSIONS: A method for compensation of time delay by modelling it as transportation lag has been presented. The classical linear-quadratic optimal control has been used. The resulting control law utilizes the response of the system as well as the past control forces. It has been shown that significant response reduction can be obtained by using this technique. Also, there is not much increase in control energy with increase in time delay. This technique ensures the stability of the controlled structure as well as the desired response reduction.

5. REFERENCES:

[1] Agrawal, A.K., Fujini, Y. and and Bhartia, B.K.(1992), Trans. Japan National Symposium on Active Structural Control, 33-44.

[2] Hiratsuka, S. and Ichikawa, A.(1969), IEEE Trans. Autom. Control AC-14, No. 3, 237-247.

[3] Kirk, D.E., " Optimal control theory ", Prentice - Hall, Englewood Cliffs, New Jersey, 1970.