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FORMULATION OF NONLINEAR DYNAMICS OF CABLE-STRUCTURE SYSTEM BY GLOBAL/LOCAL MODE APPROACH

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Introduction: In common practice of dynamic analysis, the cable-structure systems are modelled by 3D-finite element model where cables are simplified by equivalent tendon elements and thus *global* vibration is obtained. *Local* vibration of cable, on the other hand, is obtained as vibration of cable under fixed anchorages (Fig. 1). This separate treatment of local and global vibrations ignores the interaction between them. The interaction, however, is shown to be significant (Ref. 1). It appears as internal resonance; e.g., global vibration can excite cable local vibration resonantly, and *vice versa.* Investigation of the internal resonance can be implemented using the 3D-finite element model by discretizing each cable into small elements. However, number of cables in a structure are often many and hence numerical burden increases in an intolerance level even under sophisticated computer.

This paper presents an alternative, *global/local mode approach*; i.e., total motions of the system can be expressed in terms of global and local motions. The global motions are 3-D motions of the structure including quasi-static motions of the cables due to their support movements and the local motions are the rest. Using Lagrange's formulation, governing equations of global and local modes of a system with small sag cable are obtained. The equations show that, at some frequency tunings, internal resonance can be potentially induced through local-local and/or global-local couplings. Only the modes related to these frequency tunings should be selected and employed in the investigation of cable-structure dynamics. Therefore, the number of degree-of-freedoms to be solved is small.

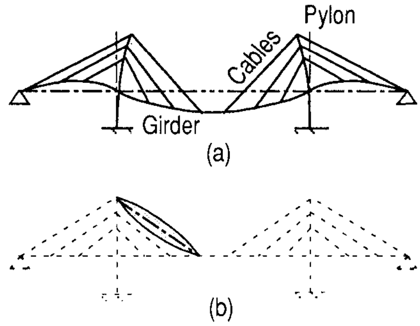

Fig. 1. Schematic drawing of (a) Global vibration; and (b) local vibration using a cable-stayed bridge.

Separation of cable motions: 3-D dynamic motions of cable (u_L, v_L and w_L) are expressed in local Cartesian coordinate (Fig. 2). They can be separated into two parts, quasi-static motions (denoted by superscript (q)) and purely dynamic motions.

The quasi-static motions are the displacements of cable which moves as an elastic tendon due to support movements. They satisfy the time-dependent boundaries statically. The purely dynamic motions can be treated by conventional procedure for cable with fixed ends, i.e., separation-of-variables method. The dynamical cable motions can be expressed as

$$u_L(x,t) = u_L^{(q)}(x,t) \tag{1}$$

$$v_L(x,t) = v_L^{(q)}(x,t) + \sum_n \phi_n(x) y_n(t) \tag{2}$$

$$w_L(x,t) = w_L^{(q)}(x,t) + \sum_n \psi_n(x) z_n(t) \tag{3}$$

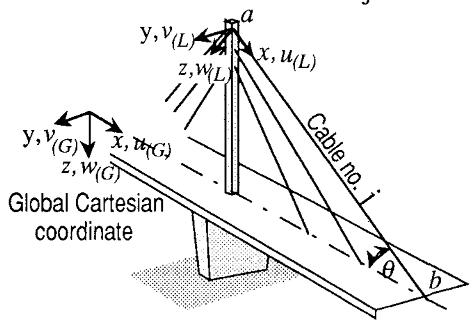

Fig. 2. Local and global Cartesian coordinate system

where y_n and z_n are generalized coordinates for out-of-plane and in-plane motions. Out-of-plane and in-plane linear undamped mode shapes of cable with fixed ends can be employed for the spatial functions, ϕ_n and ψ_n.

Global and local modes: Global motions (u_G, v_G and w_G) are expressed in global Cartesian coordinate. They consist of structural motions and quasi-static motions of cables. Using separation-of-variables method, the global motions can be expressed in terms of global generalized coordinates (q_r) and components of global mode (Φ_r^u, Φ_r^v and Φ_r^w).

$$\begin{Bmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{Bmatrix}_{(G)} = \sum_r q_r(t) \begin{Bmatrix} \Phi_r^u(x,y,z) \\ \Phi_r^v(x,y,z) \\ \Phi_r^w(x,y,z) \end{Bmatrix}_{(G)} \quad (4)$$

The global modes may be the eigen-modes computed by 3D-FEM where cables are treated as tendons in the formulation. The effect of initial stresses should be included in the formulation. Local motions are the last term on the right hand side of Eqs. 2 and 3, i.e., the purely dynamic motions of each cable. Thus, the spatial functions (ϕ_n and ψ_n) are local modes of the system. Interaction between the global and local modes is taken into account through the motions at cable anchorages.

Application of global/local mode approach to a cable-structure system: The global/local mode approach is employed in this section to obtain algebraic governing equations of a cable-structure system. Derivation is based on metallic cable normally used in engineering practice. Finite cable motions are considered. The cable sag is assumed to be small. Motions at the cable supports are also small. Proportional damping is assumed. The global modes are assumed to be orthogonal.

Firstly, the local and global mode shapes are obtained; the local mode shapes are obtained from the linear undamped model of cable (Ref. 2) and the global mode shapes from conventional FEM. The quasi-static motions are expressed in terms of the motions at cable anchorages. Then, Lagrange formulation is employed and the governing equations are obtained as (Ref. 3):

For k^{th} global mode

$$M_k\left[\ddot{q}_k + 2\xi_k\omega_k\dot{q}_k + \omega_k^2 q_k\right] + \sum_j\sum_n\left[R_{kn}\ddot{y}_n + S_{kn}\ddot{z}_n\right]_{(j)} + \sum_j\sum_n\left[Q_{kn}(y_n^2 + z_n^2)\right]_{(j)} = F_{qk} \quad (5)$$

For n^{th} out-of-plane local mode of j^{th} cable:

$$\left\{\begin{aligned} & m_{yn}\left(\ddot{y}_n + 2\xi_{yn}\omega_{yn}\dot{y}_n + \omega_{yn}^2 y_n\right) + \sum_k \nu_{nk} y_n (y_k^2 + z_k^2) \\ & + \sum_k 2\beta_{nk} y_n z_n + \sum_r 2Q_{rn} q_r y_n + \sum_r R_{rn}\ddot{q}_r = F_{yn} \end{aligned}\right\}_{(j)} \quad (6)$$

For n^{th} in-plane local mode of j^{th} cable:

$$\left\{\begin{aligned} & m_{zn}\left(\ddot{z}_n + 2\xi_{zn}\omega_{zn}\dot{z}_n + \omega_{zn}^2 z_n\right) + \sum_k \nu_{nk} z_n (y_k^2 + z_k^2) \\ & + \sum_k 2\beta_{nk} z_n z_k + \sum_k \beta_{kn} (y_k^2 + z_k^2) \\ & + \sum_r 2Q_{rn} q_r z_n + \sum_r S_{rn}\ddot{q}_r = F_{zn} \end{aligned}\right\}_{(j)} \quad (7)$$

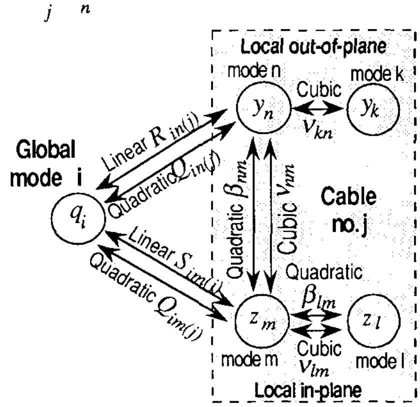

Fig. 3. Topology of global-local and local -local couplings in arbitrary chosen modes

where M_k, m_{yn} and m_{zn} are generalized modal masses; ξ_k, ξ_{yn} and ξ_{zn} are modal damping ratios; ω_k, ω_{yn} and ω_{zn} are modal frequencies; q_k, y_n and z_n are generalized coordinates; Q_{kn}, R_{kn} and S_{kn} are coefficients of coupling between global and local modes; α_{kn}, β_{kn} and ν_{kn} are coefficients of coupling between local modes; F_{qk}, F_{yn} and F_{zn} are generalized forces.

Internal resonance: Governing equations of local and global modes show that internal resonance is possible to occur. Interaction between local modes is possible through quadratic and cubic couplings; i.e., internal resonance can be potentially induced with some frequency ratios such as 1:3, 1:2, 2:1 and 3:1 (Ref. 4). Interaction between local and global modes appears as linear and quadratic couplings and internal resonance can be potentially induced with some frequency ratios such as 1:1, 1:2 and 2:1. Topology of all possible couplings is shown in Fig. 3. By investigating the frequency ratios, the modes whose frequencies are linearly and nonlinearly tuned can be identified and employed in dynamic analysis of cable-structure system. Thus, the number of degree-of-freedoms to be solved is small. Warnitchai et al. (Ref. 5) conducted a dynamic experiment on interaction of local/global motions using a small cable-stayed beam model. Not only linear internal resonance, but also nonlinear auto-parametric resonance in the model was identified in their experiment.

References: 1) A. M. Abdel-Ghaffar and M. A. Khalifa 1991 *ASCE J. of Eng. Mech.* **117**(11), 2571-2589. 2) H.M. Irvine 1981 *Cable Structures.* 3) P. Warnitchai, Y. Fujino and T. Susumpow 1992 (Preparation) *J. of Sound and Vibration.* A nonlinear dynamical model of cable and its application to a cable-structure system. 4) A.H. Nayfeh and D.T. Mook 1979 *Nonlinear Oscillations.* 5) Y. Fujino, P. Warnitchai and B.M. Pacheco 1991 *Proc. of JSCE* **No.432/I-16**, 109-118. (in Japanese).