I-623 FLEXURAL-TORSIONAL BUCKLING ANALYSIS OF LONG-SPAN CABLE-STAYED BRIDGES UNDER WIND LOAD

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1. INTRODUCTION

When a cable-stayed bridge has a longer span, elastic stability of the whole bridge increases its importance in design due to the larger axial forces in the bridge deck and the towers under gravity load, as well as the larger lateral bending moments in the bridge deck under static wind load. However, previous research on the static instability of cable-stayed bridges has generally been directed toward the bending (in-plane) buckling of the whole system under gravity load.

In this paper, linearized flexural-torsional (out-of-plane) buckling analysis of the whole system of a long-span cable-stayed bridge under static wind load is presented.

2. FORMULATION OF CONSISTENT GEOMETRIC STIFFNESS MATRIX

The incremental virtual work equation of equilibrium relating an increment of applied force to the increment of corresponding displacement can be expressed, for a three-dimensional beam, as

$$\int_V \left[E e_{xx} \delta e_{xx} + 4G e_{yx} \delta e_{yx} + 4G e_{zx} \delta e_{zx} \right] dV + \int_V \left[\sigma_x \delta \eta_{xx} + 2\tau_{yx} \delta \eta_{yx} + 2\tau_{zx} \delta \eta_{zx} \right] dV = {}^2R - {}^1R \tag{1}$$

in which E = modulus of elasticity; G = shear modulus of elasticity; e_{xx}, e_{yx}, e_{zx} and η_{xx}, η_{yx}, η_{zx} denote the linear and nonlinear components of the incremental Green-Lagrange strains; σ_x, τ_{yx}, τ_{zx} denote Cauchy stresses; ${}^2R$ = the external work; ${}^1R$ = internal resisting work; and V = volume of the beam. Then, the incremental equilibrium equation in the updated Langrangian formulation is obtained after lengthy calculations on Eq. 1 as

$$[k_e]\{u\} + [k_g]\{u\} = \{{}^2f\} - \{{}^1f\} \tag{2}$$

in which $[k_e]$ = the linear elastic stiffness matrix; $[k_g]$ = the geometric stiffness matrix, which is given in reference 1 for a three-dimensional beam with bi-symmetric cross-section and neglecting warping effect; {u} = the incremental displacement vector; $\{{}^2f\}$ and $\{{}^1f\}$ are the element nodal external and internal forces. The $[k_g]$ in Eq. 2 is taken into account the effects of initial axial force, bending moment, and torsion.

For out-of-plane buckling analysis, the $[k_g]$ in Eq. 2 has to be corrected to represent the generalized rotation displacements that maintain continuity at angle joints of structures after out-of-plane deformation, and then the resulting matrix may be expressed as

$$[k_{gc}] = [k_g] + [k_j] \tag{3}$$

in which $[k_{gc}]$ and $[k_j]$ may be called consistent geometric stiffness matrix and joint-rotation correction matrix given in reference 1, respectively.

3. BUCKLING ANALYSIS UNDER STATIC WIND LOAD

The algorithm to compute a critical wind velocity caused buckling in linearized buckling analysis under nonproportional loads is summarized as follows: (1) Perform static analysis for the two given loading conditions, namely, gravity load and wind load; (2) compute the consistent geometric stiffness matrices of the structure for the gravity load $[K_{gcG}]$ and wind load $[K_{gcW}]$; (3) impose consistent geometric stiffness matrix of the structure for gravity load on linear elastic stiffness matrix of the structure $[K_e]$, and perform the equilibrium equations for the nodal degree of freedoms with load $(\{{}^2F\}-\{{}^1F\})=0$:

$$\Big(\big([K_e] + [K_{gcG}] \big) + \lambda [K_{gcW}] \Big) \{U\} = 0 \tag{4}$$

in which {U} is the vector of nodal displacements, and the constant λ is a scale factor that multiplies initial wind load; (4) determine eigenvalues and eigenvectors of above equation; and (5) compute the critical wind velocity from $V_c = \sqrt{\lambda} V_0$ where V_0 is the design wind

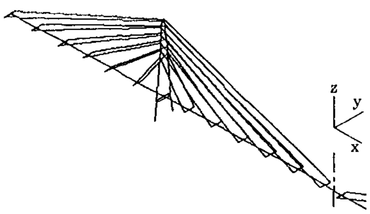

Fig.1 A three-dimensional finite element bridge model.

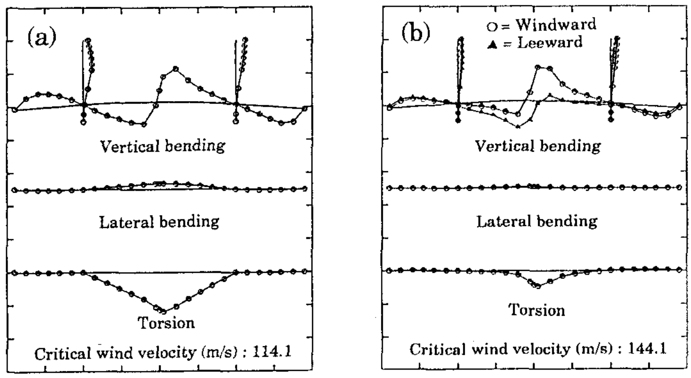

Fig.2 Buckling modes and corresponding critical wind velocities under load $(D+P_S)+\lambda W$ using : (a) 2-D model, (b) 3-D model.

velocity.

4. NUMERICAL EXAMPLES

The linearized instability analysis of a cable-stayed bridge with a three-span continuous deck, a 1000 m center span with two 450 m side spans[1], has been studied under static wind load. Two-dimensional (2-D) and three-dimensional (3-D) bridge models shown in **Fig.1** are used for comparisons. The results of buckling analysis shown in **Table 1** indicate that a critical wind velocity obtained from 2-D model under load (D + Ps) + λW is about 26 percent lower than that obtained from 3-D model. This results from neglecting the torsional stiffness contribution of double-plane stay cables in 2-D model. Consequently, the 2-D model appears to be conservative in predicting a critical wind velocity for the flexural-torsional buckling, compared with the 3-D model. When effects of live load are included in the analysis, only slight reduction in critical wind velocity is obtained (**Table 1**). This is probably due to high buckling load factor for gravity load, and the live load is low in comparison with the dead load.

Table 1 Buckling comparisons obtained from 2-D and 3-D models under static wind load.

Combined load description	Buckling mode description	Critical wind velocity (m/s) 2-D model	Critical wind velocity (m/s) 3-D model
(D + Ps) + λW	VB + LB + T	114.1	144.1
(D + Ps + L) + λW	VB + LB + T	112.9	not considered

Legend: W - Wind load; D, L - Dead and live loads; Ps - Prestress; LB - Lateral bending; VB - Vertical bending; T - Torsion

The first buckling modes and corresponding load factors obtained from the 2-D and 3-D models under load (D + Ps) + λW are illustrated in **Fig.2a** and **2b**, respectively. As can be seen from these figures, the buckling mode under static wind load is flexural-torsional mode. The vertical bending mode is asymmetric while the torsional and lateral bending modes are symmetric. The phenomenon of asymmetric vertical bending mode is probably due to the higher stiffness of upper cables which are back-anchored at side supports. It should also be noted that because of unsymmetric supports at ends of bridge deck (hinge support at one end and roller support at the other), the vertical bending mode deviates slightly form asymmetric mode.

5. CONCLUSIONS

Linearized flexural-torsional buckling analysis of a long-span cable-stayed bridge under static wind load has been presented. The results of analysis indicate that buckling of a long-span cable-stayed bridge under static wind load is predominantly a combined flexural and torsional mode. Further study should be extended to nonlinear instability analysis of long-span cable-stayed bridges under displacement-dependent static wind load, especially lift force and pitching moment, together with the effect of lateral drag force.

6. REFERENCE : 1) Boonyapinyo. V., Yamada. H., and Miyata. T., 'In-Plane and Out-of-Plane Buckling Analysis of a Long-Span Cable-Stayed Bridge,' will be published in Structural Eng. / Earthquake Eng., Proc. of JSCE.