## I-619

## INITIATION OF PROPAGATING BUCKLES FROM LOCAL DAMAGE IN PIPELINE

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1. INTRODUCTION: This paper studies the initiation of a propagating buckle in a deep-water pipeline. If the external pressure is high enough, then a propagating buckle can be initiated by a local damage in the pipe. The initiation pressure,  $P_i$ , is the minimum pressure at which a local dent would grow and transform itself into a propagating buckles. A satisfactory evaluation for  $P_i$  is important for the offshore industry to judge whether a given damage on a pipeline has the potential of being an initiation point for a propagating buckle. Based on the finite element technique accounting for geometric and material nonlinearities, a three-dimensional simulation of the common experimental procedure is conducted to create a local damage in a pipe. Then, the technique is applied to obtain the initiation pressure in the deep-water pipeline. Nine-node isoparametric shell finite elements are equipped with the pressure node [2] which facilitates the tracing of unstable equilibrium paths. The result of the analysis is compared with those of experiments [1].

2. FINITE ELEMENT METHOD: The analytical technique employed in this work is based on the Ahmad type degenerated shell element which is adapted to nonlinear analysis. The formulation conducted here uses an updated Lagrangian formulation with a net of convected coordinate lines embedded in and deforming with a pipe. An equilibrium of the deformed state is ensured through fulfillment of the principle of virtual work, which, in the absence of body forces, takes the form:

$$\int_{V} \delta U_{k,j} (\mathbf{G}^{k} \cdot \mathbf{g}_{i}) \dot{\tau}^{ij} dV + \int_{V} \delta U_{k,j} (\mathbf{G}^{k} \cdot \dot{\mathbf{g}}_{i}) \tau^{ij} dV$$

$$= \int_{b} \delta u_{i} \dot{T}^{i} db - \left( \int_{V} \delta U_{k,j} (\mathbf{G}^{k} \cdot \mathbf{g}_{i}) \tau^{ij} dV - \int_{b} \delta u_{i} T^{i} db \right) \tag{1}$$

in which  $G^k$  is contravariant base vectors in the reference configuration and  $\mathbf{g}_i$  is covariant base vectors in the current configuration. The metric tensors  $G_{ij}$  and  $g_{ij}$  can be obtained by  $G_i \cdot G_j = G_{ij}$  and  $\mathbf{g}_i \cdot \mathbf{g}_j = g_{ij}$ , and  $G^{ij}$  and  $g^{ij}$  are their respective inverse. Then, we can write  $G_i = G_{ij}G^j$ ,  $\mathbf{g}_i = g_{ij}\mathbf{g}^j$ ,  $\mathbf{G}^i = G^{ij}\mathbf{G}_j$ , and  $\mathbf{g}^i = g^{ij}\mathbf{g}_j$ . In (1) Kirchhoff stress tensor  $\tau^{ij}$  is defined as  $\tau^{ij} = \sqrt{\frac{g}{G}}\sigma^{ij}$ , where G and g denote the determinants of metric tensors  $G_{ij}$  and  $g_{ij}$  respectively, and  $T^i$  is defined as tractions in the current boundary b.

In this work, the finite-strain, elastoplastic constitutive equations based on J<sub>2</sub>-flow theory of plasticity with isotropic hardening are considered. Let Y and s denote the so-called von Mises stress and the deviatoric Kirchhoff stress tensor, respectively, i.e.,  $Y = \sqrt{(3J_2)} = \sqrt{\left(\frac{3}{2}g_{ik}g_{jl}s^{ij}s^{kl}\right)}$  and  $s^{ij} =$ 

 $\tau^{ij} - \frac{1}{3}g^{ij}g_{kl}\tau^{kl}$  ( $J_2$  is the second invariant of s). Denoting  $\hat{\tau}$  as the Jaumann rate of Kirchhoff stress and d as the rate of deformation, a finite-strain version of  $J_2$  elastoplasticity can be represented by

$$\begin{split} & \stackrel{\Delta}{\boldsymbol{\tau}} &= \quad \mathcal{E} : (\mathbf{d} - \mathbf{d}^p) \;, \\ \mathcal{E}^{ijkl} &= \quad \frac{E}{2(1+\nu)} \left[ (g^{ik}g^{jl} + g^{il}g^{jk}) + \frac{2\nu}{1-2\nu}g^{ij}g^{kl} \right] \;, \\ & \mathbf{d}^p \; = \; \dot{d}^p \frac{\partial \varphi(\tau, Y_{max})}{\partial \tau} \;, \\ & \dot{Y}_{max} \; = \; \frac{d \, Y_{max}}{d \, d^p} \dot{d}^p \;. \end{split}$$

Here,  $d^p$  denotes the rate of effective plastic strain, i.e.,  $\dot{d}^p = \sqrt{\left(\frac{2}{3}d^p:d^p\right)}$ and  $\varphi(\tau, Y_{max}) = Y - Y_{max}^{\tau}$  is yield function and the function  $\frac{dY_{max}}{dd^p}$  defines the hardening law. Finally, the loading/unloading conditions can be formulated in the standard Kuhn-Tucker  $\varphi \leq 0, \ \dot{d}^p \geq 0, \ \dot{d}^p \varphi = 0.$ 

By introducing an additional equation of volume change due to the deformation from the current state to the nearby state, i.e.,

 $\Delta \hat{V} = \int \Delta \mathbf{u} \cdot \mathbf{n} \ db^e \,,$ (2)

(n denotes outward unit vector normal to the boundary  $b^e$ ) the pressure node, which traces equilibrium paths around limit points under uniform change of pressure, is implemented (see [2] for modified element stiffness matrix). Thus, along unstable equilibrium paths, we can specify the change of the volume to solve the required change of pressure. If several or all elements are subjected to the same

change of pressure, the pressure node can be shared.

3. RESULTS: Results are presented for an aluminum-alloy (Al-6061-T6) pipe with outside diameter D=1.250~in, thickness t=0.035~in (D/t=35.7) and initial imperfection 0.008D. The material properties are given in [2]. A locally damaged segment of the pipe, under external pressure, can lead to the initiation of a buckle that propagates along the pipe. Thus, the first step towards the propagating buckle analysis is to inflict initial local damage somewhere along the pipe. The computations necessary for this step were carried out with a segment of the pipe of length equal to 4 diameters. Since the present study deals exclusively with damages which are symmetric, only one quarter of the pipe was analyzed. A local damage was obtained by applying uniform pressure on the 3 elements of the ring by means of the pressure node common to these elements. The equilibrium path for this loading was traced by imposing decrements of the volume associated with the pressure

Fig. 1 presents a plot of the pressure, p, applied on the 3 elements, versus the reduction of volume enclosed by the pipe,  $\Delta V$ , normalized by the volume enclosed in the undeformed configuration,  $V_o$ . The pressure reaches a critical value at the limit point of the equilibrium path, then decreases slowly with considerable additional volume reduction, reaches a minimum value and increases slightly to the point where the initial local damage is ruled substantial enough. Then, the pressure on the pipe was removed completely. Of course, considerable residual volume reduction remained after unloading. Fig. 1 shows also the resulting damage on 8D pipe just after complete pressure removal.

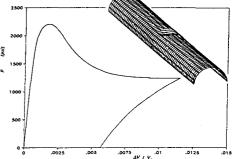


Fig.1. Initial Local Damage

It was found that the geometry of local damage can be well represented by  $D_{min}/D_{max}$  where  $D_{min}$  and  $D_{max}$  are the minimum and maximum diameters of the most damaged section [1]. From fig. 1,  $D_{min}/D_{max}=0.7425$  is obtained. With the initial local damage in place on the 4D segment, the analysis entered the phase of initiation of the propagating buckle. A 24D pipe (a new 20D pipe pasted to the existing 4D segment) was considered sufficient to idealize a long pipe for the purposes of initiation and steady-state propagation of the buckle [1]. External pressure was applied uniformly on 24D pipe by means of the pressure node common to all finite elements. As in the phase of local damage, the equilibrium path for uniform pressure on the pipe was traced by imposing decrements of the volume associated with the pressure node.

Fig. 2 presents a plot of the applied pressure, p, versus  $\Delta V/V_o$ . After the pressure reached a maximum pressure, the pipe in the neighborhood of the initial local damage began to collapse. From fig. 2, it is seen that the pressure sharply increases up to the maximum pressure with small volume reduction. The pressure-carrying capacity of the pipe fell rapidly with significant pressure reduction during collapse just after the maximum pressure.

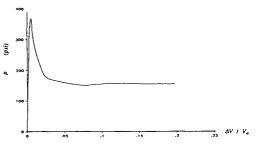


Fig. 2.  $p-\Delta V/V_o$  Curve During Propagating Buckle

Eventually, the pressure reached a minimum just before first contact, then it increased slightly again just after first contact to attain an approximately constant value, the propagation pressure,  $p_p = 156.1 \ psi$  (155.8 psi in experiment [2].) As the volume was further reduced, the propagating buckle continued to translate longitudinally. The initiation pressure, which corresponds to the maximum pressure, is obtained as  $p_i = 369 \ psi$ . This is close agreement with experimental value,  $p_i = 381 \ psi$ , obtained from the local damage of  $D_{min}/D_{max} = 0.7425$ .

4. CONCLUSION: A finite element technique is applied for the initiation of propagating buckle. Based on the updated Lagrangian formulation in the convected coordinate system, nine node isoparametric finite elements are employed with the pressure node. Its advantages over existing analytical tools include the capability of simulating three-dimensional pipes of arbitrary thickness and the volume control by the pressure node, which facilitates the transition over limit pressure points.

5. REFERENCES: [1] Kyriakides, S., et. al., "Initiation of Propagating Buckles from Local Pipeline Damages," Journal of Energy Resources Technology, Vol. 106, pp. 79–87, 1984.

[2] Song, H.-W.; Propagating Buckle Analysis of Deep-water Pipelines, 46th Annual Conference of JSCE, Osaka, Sept. 1991.