I-618

# Analysis of an Embedded Circular Arch

Niigata Univ. Student Member Jerson New Orejudos
Niigata Univ. Member Hideo Ohkawa

## 1. Introduction

In line with the objective of optimizing land space due to the ever-increasing world population, use of an underground housing system can aptly be advocated. Similarly, with the advent of wide-ranging sea explorations, quasi-permanent underwater shelters in the site can radically improve mobilization. These human shelters may be enclosed by a dome structure composed of, or supported by, intersecting circular arches. Analysis of the dome, therefore, requires the fundamental analysis of an embedded (i.e., underwater or underground) circular arch, which is the foremost objective of this study. Using the classical Castigliano's theorem, analysis of an embedded two-hinged circular arch is conducted. The analysis superposes loadings due to the weight of the surrounding medium and the arch itself and the horizontal component of the hydrostatic forces as shown in Fig. (1). General formulation for reactions, internal forces (moment, shear and axial forces) and displacements are derived and a numerical analysis for support angle $\theta=90^{o}$ is done with appropriate graphs shown in their entirety.

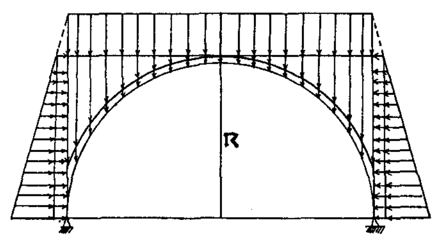

**Fig. (1) Load Idealization**

## 2. Horizontal Redundant Reaction

Using Castigliano's theorem, the horizontal redundant reaction at the right support can be found as:

$$H=-\frac{\frac{R}{EI}\int_{-\theta}^{\theta}M'(-f+R-R\cos\phi)d\phi+\frac{R}{AE}\int_{-\theta}^{\theta}N'\cos\phi d\phi+\frac{R}{GFA}\int_{-\theta}^{\theta}V'\sin\phi d\phi}{\frac{R}{AE}\int_{-\theta}^{\theta}\cos^{2}\phi d\phi+\frac{R}{EI}\int_{-\theta}^{\theta}\left(f^{2}-2fR+R^{2}+2fR\cos\phi-2R^{2}\cos\phi+R^{2}\cos^{2}\phi\right)d\phi+\frac{R}{GFA}\int_{-\theta}^{\theta}\sin^{2}\phi d\phi}$$

where: A=cross-sectional area; E=modulus of elasticity; F=shape factor=5/6 for a rectangular cross section; f=rise of the arch; G=shear modulus of elasticity; I=moment of inertia=$bt^3/12$ for a rectangular cross-section; M', V' & N'=bending moment, shear, and axial force, respectively, of the primary arch; R=radius of curvature; $\phi$=angle of any arbitrary point along the arch measured from the vertical, and $\theta$=support angle.

A numerical analysis for a submerged arch has been thoroughly done considering the following data: Material: AISI 1025 Steel; $A=0.03m^2$; b=0.15m; D=40m; $E=2X10^{11}N/m^2$; F=5/6; $G=7.612X10^{10}N/m^2$; $\gamma a=7.7X104N/m^3$; $\gamma=9810N/m^3$; $I=1X10^4m^4$; R=10m, and t=0.20m. Fig. 2 shows the distribution of H.

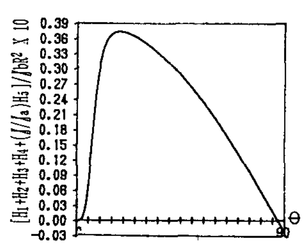

**Fig.(2) Distribution of H**

## 3. Moment, Shear and Axial Forces

Figs. (3), (4), and (5) show the distribution of moment shear and axial forces, respectively, for arch of angular support $\theta=90^{o}$.

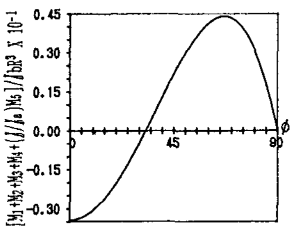

Fig. (3) Moment Distribution

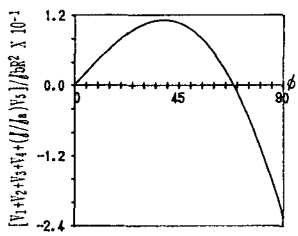

Fig. (4) Shear Distribution

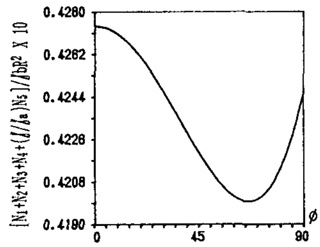

Fig.(5) Axial Force Distribution

## 4. Deformation

Fig. (6) shows the final deformed shape of the 90° arch.

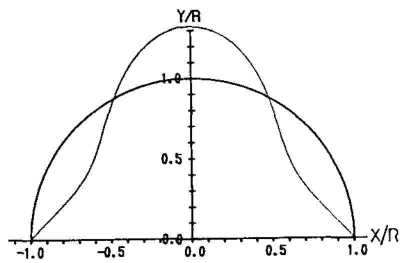

Fig. (6) Final Deformed Shape Exaggerated 100X

## 5. Conclusion and Recommendation

The foregoing study is the most fundamental part in the analysis of an embedded arch, which may serve as an integral supporting section or member of an embedded dome. As such, it is recommended that the following areas will be considered in future studies:

- consideration of dynamic loading, e.g., wave motion, ground vibration, etc.;
- consideration of static loading but non-linear deformation;
- consideration of dynamic loading and non-linear deformation, and
- integration of intersecting arches towards the formation of a structural dome.

As Castigliano's theorem is applicable only to the elastic deformation of the arch, it is expected that other appropriate methods will be used when non-linearity will be considered. Likewise, analysis from two-dimensional will have to be shifted to three-dimensional by the time a structural dome will be considered.

## 6. References

1. Orejudos, J. and Ohkawa, H., "Analysis of an Embedded Circular Arch:Hydrostatically Distributed Horizontal Loading", Proceedings from the 19th Conference of the Japan Society of Civil Engineers, Kanto Chapter, pp. 26-27, March, 1992.
2. Orejudos, Jerson, "Analysis of an Embedded Circular Arch", A thesis presented for the Degree of Master of Engineering, Niigata University, Feb., 1992.
3. Orejudos, J. and Ohkawa, H., "Analysis of an Embedded Circular Arch:Vertical Loading Condition", Proceedings from the 9th Conference of the Japan Society of Civil Engineers, Niigata Chapter, pp.19-24, Oct., 1991.
4. Orejudos, J. and Ohkawa, H., "Static Analysis of an Embedded Symmetrical Two-Hinged Circular Arch Under Vertical Loading Condition", Proceedings from the 46th National Conference of the Japan Society of Civil Engineers, Part I, pp. 126-127, September, 1991.
5. Timoshenko, S.P. and Young, D.H., Theory of Structures, McGraw-Hill Book Co., New York, 1965.