

# A MULTIAXIAL CYCLIC PLASTICITY MODEL FOR STEELS WITH YIELD PLATEAU

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## 1. INTRODUCTION

In the present paper, a uniaxial two-surface model proposed by authors in Ref. 1) is extended to multiaxial stress state. Furthermore, the comparison between the experiment and prediction by the present model is given.

## 2. THE PROPOSED MULTIAXIAL TWO-SURFACE MODEL

### 1) Loading and Bounding Surface

The Von Mises type of loading surface adopted in the present paper is:

$$f(\sigma_{ij}, \alpha_{ij}, \kappa) = \frac{3}{2} \{ (S_{ij} - \alpha_{ij})(\bar{S}_{ij} - \alpha_{ij}) \}^2 - \kappa^2 = 0 \quad (1)$$

Similarly, the bounding surface is:

$$F(\bar{\sigma}_{ij}, \beta_{ij}, \bar{\kappa}) = \frac{3}{2} \{ (\bar{S}_{ij} - \beta_{ij})(\bar{S}_{ij} - \beta_{ij}) \}^2 - \bar{\kappa}^2 = 0 \quad (2)$$

where  $\sigma_{ij}$  and  $S_{ij}$  are the stress and deviatoric stress components on the loading surface while  $\bar{\sigma}_{ij}$  and  $\bar{S}_{ij}$  are on the bounding surface;  $\kappa$  and  $\bar{\kappa}$  represent the radii of the loading and bounding surfaces respectively;  $\alpha_{ij}$  and  $\beta_{ij}$  indicate the center of the two surfaces (see Fig. 1).

### 2) The Motion of Two Surfaces

Based on the uniaxial two-surface model in Ref. 1), the motion of the bounding surface can be defined as follows.

$$d\beta_{ij} = \frac{2}{3} E_0^P d\varepsilon_{ij}^P \quad (3)$$

where  $E_0^P$  is the slope of the bounding line in the uniaxial case;  $d\varepsilon_{ij}^P$  represents the increment of plastic strain. In the present model, it is assumed that the centers of the two surfaces and the loading point always keep on one line during the motion (see Fig. 1). As the result, there exists the following relation between the loading and bounding surface (see Fig. 1)

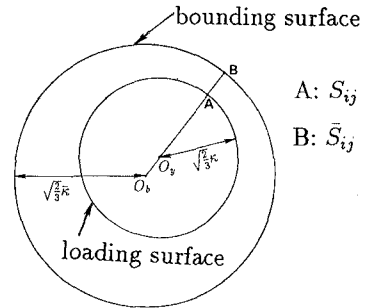
$$S_{ij} - \beta_{ij} = \Gamma (S_{ij} - (\alpha_{ij} + d\alpha_{ij})) \quad (4)$$

where  $\Gamma$  is a scalar and expressed as follows.

$$\Gamma = \begin{cases} \frac{3}{2} \|S_{ij} - \beta_{ij}\|^2 / \kappa^2 & \text{if } (S_{ij} - \beta_{ij})(S_{ij} - \alpha_{ij}) \geq 0 \\ -\frac{3}{2} \|S_{ij} - \beta_{ij}\|^2 / \kappa^2 & \text{otherwise} \end{cases}$$

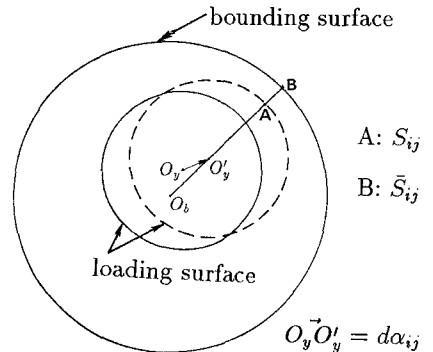
Therefore, the motion of the loading surface is obtained.

$$d\alpha_{ij} = (S_{ij} - \alpha_{ij}) - (S_{ij} - \beta_{ij}) / \Gamma \quad (5)$$



$O_y$ :  $\alpha_{ij}$ , center of the loading surface  
 $O_b$ :  $\beta_{ij}$ , center of the bounding surface

Fig. 1 Loading and bounding surfaces



$O_y$ :  $\alpha_{ij}$ , center of the loading surface  
 $O_b$ :  $\beta_{ij}$ , center of the bounding surface  
 $O'_y$ : center of the new loading surface

Fig. 2 Motion of the two surfaces

3) Calculation of the Plastic Modulus in Constitutive Equation

In the present model, the plastic modulus  $E_0^P$  is assumed to have the same expression as in the uniaxial two-surface model in Ref. 1), i.e.,

$$E^P = E_0^P + h \frac{\delta}{\delta_{in} - \delta} \tag{6}$$

where  $\delta$  is defined as the distance between the two surfaces along the direction  $(S_{ij} - \beta_{ij})$ (as shown in Fig. 2); the parameters  $E_0^P$ ,  $h$  and  $\delta_{in}$  have the same meaning as in the uniaxial model in Ref. 1). It can be known that when the two surfaces contact,  $\delta$  will be equal to zero and the two surfaces are tangential to each other.

3. MODIFICATION OF THE EQUATIONS CONCERNING THE ACCUMULATED EFFECTIVE PLASTIC STRAIN

In order to extend the uniaxial model to the multiaxial case easily, the equations in Ref. 1) concerning the accumulated effective plastic strain(as abbreviated to A.E.P.S.) are modified. For example, A.E.P.S. used in the calculation the elastic range(i.e. Eq.(6) in Ref. 1) is replaced by the accumulated plastic strain  $\tilde{\epsilon}^P$ , which can be expressed as follows.

$$\tilde{\epsilon}^P = \int d\epsilon^P = \int \sqrt{\frac{2}{3} d\epsilon_{ij}^P d\epsilon_{ij}^P} \tag{7}$$

While the A.E.P.S. used to calculate the end of yield plateau(i.e., Eq.(9) in Ref. 1)) is substituted by the maximum plastic strain(M.P.S.)  $\tilde{\epsilon}_{max}^P$  as shown in Eq. (8).

$$\tilde{\epsilon}_{max}^P = \left\{ \sqrt{\frac{2}{3} \epsilon_{ij}^P \epsilon_{ij}^P} \right\}_{max} \tag{8}$$

In the calculation of the bounding surface radius, the following equation is adopted for the loading path after the  $i$ -th reversal loading point.

$$\bar{\kappa}_i = \bar{\kappa}_\infty + (\bar{\kappa}_0 - \bar{\kappa}_\infty) \exp(-\zeta W_i^P) \tag{9}$$

where  $\bar{\kappa}_0$  is the radius of the initial bounding lines;  $\zeta$  and  $\bar{\kappa}_\infty$  are the constants. They are determined from one loading cycle experiment.

5. EXAMPLES

The proposed model has been implemented with finite element method based on the program FEAP. The specimen shown in Fig. 3 is used to examine the applicability of the present model, which is loaded in longitudinal direction cyclically. Since the section of the specimen is not uniform, there exists multiaxial stress state in it. The comparison between the experiment and prediction with the proposed model is shown in Fig. 4. It can be seen that a good agreement between the experiment and prediction has been obtained.

REFERENCES

1) Shen C., Tanaka, Y., Mizuno, E. and Usami T. : A two-surface model for steels with yield plateau, Proc. of JSCE, Structural Eng./Earthquake Eng. Vol.8, No.4, 179s-188s, Jan. 1992.

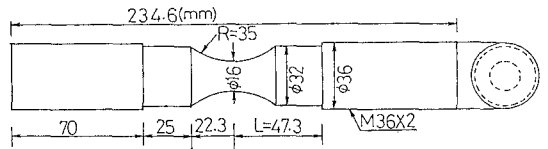


Fig. 3 The specimen in the experiment

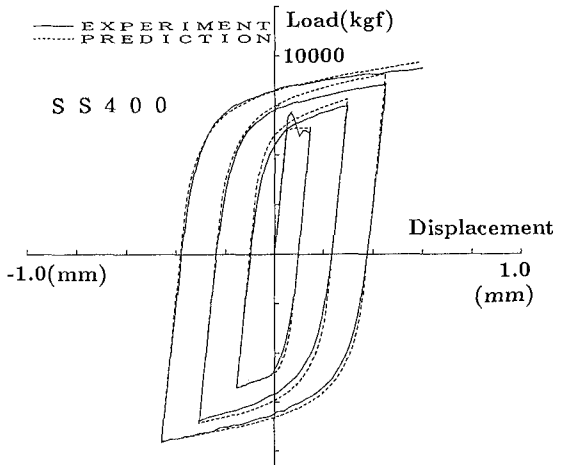


Fig. 4 The comparison between the experiment and prediction